3. *A Remark on a Periodic Boundary Problem of Parabolic Type*

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Let $Q=\{(t, x):\ -\infty<t<+\infty,\ x=(x_1, \cdots, x_n)\in \Omega\}$ and $\partial Q=\{(t, x):\ -\infty<t<+\infty,\ x\in\partial\Omega\}$, where Ω is a bounded domain in euclidean n-space E^n with boundary $\partial\Omega$. Let there be given the system of semilinear parabolic equations

(1) $\mathcal{L}_i(u_i)=f_i(t, x, u_1, \cdots, u_N)$ in Q $(i=1, \cdots, N)$

and the boundary condition

(2) $u_i=\varphi_i(t, x)$ on ∂Q $(i=1, \cdots, N)$,

where

$$\mathcal{L}_i=\frac{\partial}{\partial t}-\sum_{p,q=1}^{n}\frac{\partial}{\partial x_p}a_{pq}^i(t, x)\frac{\partial}{\partial x_q}+\sum_{p=1}^{n}b_p^i(t, x)\frac{\partial}{\partial x_p}$$

and the functions a_{pq}^i, b_p^i, f_i, and φ_i $(p, q=1, \cdots, n;\ i=1, \cdots, N)$ are periodic in t with period $T(T>0)$. In the present note we shall be concerned with the problem of finding a solution, periodic in t with period T, to the boundary problem (1), (2) which will be called the first periodic boundary problem of parabolic type.

We introduce the following assumptions:*)

I. There is a positive constant λ snch that, for any real vector ξ and for all $(t, x)\in\bar{Q}$,

$$\sum_{p,q=1}^{n}a_{pq}^i(t, x)\xi_p\xi_q\geqq\lambda\sum_{p=1}^{n}\xi_p^2 \qquad (i=1, \cdots, N).$$

II. $a_{pq}^i\in C^{1+\alpha}(\bar{Q})$ and $b_p^i\in C^{\alpha}(\bar{Q})$ $(0<\alpha<1)$ $(p, q=1, \cdots, n;\ i=1, \cdots, N)$.

III. The functions $f_i(t, x, z_1, \cdots, z_N)$ $(i=1, \cdots, N)$ are defined in $\mathcal{D}=\{(t, x, z_1, \cdots, z_N):\ (t, x)\in\bar{Q},\ -\infty<z_k<+\infty,\ k=1, \cdots, N\}$, are in $C^{\alpha}(\bar{Q})$ for each fixed $(z_1, \cdots, z_N)$, and satisfy the Lipschitz condition $|f_i(t, x, z_1, \cdots, z_N)-f_i(t, x, \bar{z}_1, \cdots, \bar{z}_N)|\leqq l_i\sum_{p=1}^{n}|z_p-\bar{z}_p|$ $(i=1, \cdots, N)$. Moreover, the system $\{f_i\}$ is quasi-monotone increasing in $z_1, \cdots, z_N$; that is, for each i and for $z_k\leqq\bar{z}_k$ $(k=1, \cdots, N)$, $z_i=\bar{z}_i$, the inequality

$$f_i(t, x, z_1, \cdots, z_N)\leqq f_i(t, x, \bar{z}_1, \cdots, \bar{z}_N)$$

holds.

IV. $\Omega\in A^{2+\alpha}$; $\varphi_i\in C^{2+\alpha}(\partial Q)$.

V. There exist functions $\underline{\omega}_i(t, x)$, $\overline{\omega}_i(t, x)$ $(\underline{\omega}_i\leqq\overline{\omega}_i)$ in $C^{\alpha}(\bar{Q})$

*) For the definitions of $C^{r+\alpha}(\bar{Q})$ $(r=0, 1, 2)$, $C^{2+\alpha}(\partial Q)$, and $A^{2+\alpha}$ see [2], [6].