57. *On the Strong (L) Summability of the Derived Fourier Series*

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1. In a recent paper, Borwein [1] has constructed a new method of summability for an infinite sequence $\{s_n\}$. He defines a sequence $\{s_n\}$ to be summable by the logarithmic method of summability or summable (L) to the sum s if, for x in the interval $(0, 1)$,

$$\lim_{x\to 1-0} \frac{1}{\log(1-x)} \sum_{n=1}^{\infty} \frac{s_n}{n} x^n = s. \tag{1.1}$$

It is known [3] that this method includes the Abel method. Recently K. Ishiguro [4] proved that if $\{s_n\}$ is summable by Riesz logarithmic mean of order one, it is also summable (L) to the same sum, but the converse is not true.

A series $c_0 + c_1 + c_2 + \cdots$ is said to be strongly summable $(c, 1)$ or summable $[c, 1]$ to the sum s, if

$$\sum_{\nu=0}^{n} |s_\nu - s| = o(n), \qquad \text{as } n\to\infty, \tag{1.2}$$

s_ν being the sum of the first $(\nu+1)$ terms of the series. The series is said to be strongly summable by Riesz logarithmic mean of order one or summable $[R, \log n, 1]$ to the sum s, if

$$\sum_{\nu=0}^{n} \frac{|s_\nu - s|}{\nu} = o(\log n), \qquad \text{as } n\to\infty. \tag{1.3}$$

We define an analogue for strong summability of (L) summability method as follows:

Definition. A series $\sum_{n=0}^{\infty} c_n$ with the sequence of partial sum $\{s_n\}$ is said to be summable by strong (L) summability to the sum s if

$$\sum_{\nu=1}^{\infty} \frac{x^\nu |s_\nu - s|}{\nu} = o\{\log(1-x)\}, \qquad \text{as } x\to 1 \tag{1.4}$$

for x in the open interval $(0, 1)$.

2. Suppose that the function $f(t)$ is Lebesgue integrable over the interval $(0, 2\pi)$ and periodic with period 2π. Let the Fourier series associated with function $f(t)$ be

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{1}^{\infty} A_n(t). \tag{2.1}$$

The series