56. A Duality Theorem for Locally Compact Groups. IV

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1. As a sequel of the previous articles [1]~[3], the present paper is devoted to prove the duality theorem which is same as shown in [3], for certain class of locally compact semi-direct product G of a separable closed abelian normal subgroup N and a closed subgroup K satisfying the assumptions $1\sim 4$. These class contains the motion group on $\boldsymbol{R}^n$, the n-dimensional inhomogeneous Lorentz group, and the transformation group of straight line.

We call an operator field $\boldsymbol{T}=\{T(D)\}$ over the set Ω_0 of all equivalence classes (representative $D=\{U_g^D, \mathfrak{H}^D\}$) of irreducible unitary representations of G *admissible* when

(1) $T(D)$ is a unitary operator in $\mathfrak{H}^D$ for any D in Ω_0.

(2) For any irreducible decomposition $\int D^\lambda d\nu(\lambda)$ of $D_1 \otimes D_2$ which is related by U,

$$U(T(D_1)\otimes T(D_2))U^{-1}=\int T(D^\lambda)d\nu(\lambda)\ .$$

The main proposition of this paper is as follows.

Proposition. *For any admissible operator field* $\boldsymbol{T}$, *there exists unique element* g *in* G *such that*

$$T(D)=U_g^D \qquad \textit{for any } D \textit{ in } \Omega_0\,.$$

2. [Assumption 1] G *is a regular semi-direct product in the sense of Mackey* [4].

Consider the dual group $\hat{N}$ of abelian group N, then g in G gives a transformation $g(\hat{n})$ on $\hat{N}$ defined by

$$\langle g(\hat{n}), n\rangle=\langle \hat{n}, g^{-1}ng\rangle\ ,$$

where brackets show ordinary dual relation between N and $\hat{N}$. We choose a representative $\hat{n}$ in given G-orbit L in $\hat{N}$, and let the isotropy group of $\hat{n}$ in G be $G(\hat{n})$, then $G(\hat{n})$ is a semi-direct product of N and a subgroup $K(\hat{n})$ in K.

For any irreducible unitary representation $\tau=\{W_k^\tau, \mathfrak{H}^\tau\}$ of $K(\hat{n})$ consider the representation $D(\hat{n}, \tau)$ of G induced by the representation $\{\langle \hat{n}, n\rangle W_k^\tau, \mathfrak{H}^\tau\}$ of $G(\hat{n})$ $(g=nk)$.

From Mackey's results ([4] Th. 14.1 and 2), $D(\hat{n}, \tau)$ is irreducible and determined by L and τ besides unitary equivalence, and arbitrary irreducible unitary representation of G is given in this form.