54. *Connection of Topological Fibre Bundles. II*

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In his note [2], the author defined the connection forms for an arbitrary topological fibre bundle ξ to be an element in $C^1(X_G, G)$ such that $s(\alpha a, \beta b)=a^{-1}s(\alpha, \beta)b$, where X_G is the total space of the principal bundle of ξ, and a, b are elements of G, the structure group of ξ. There, first we define the obstruction class for the existence of (topological) connection forms (of. [3]). Next we consider a relation between the topological curvature forms of complex vector bundles and their complex Chern classes (cf. [5]). We use the same notations as [2] in this note. For example, we denote

$C^1(X_G, G)_G=\{s \mid s\in C^1(X_G, G),\ s(\alpha a, \beta b)=a^{-1}s(\alpha, \beta)b,\ a, b\in G\}$,

$T^1(X_G, G)=\{s \mid s\in C^1(X_G, G),\ s(\alpha a, \beta b)=a^{-1}s(\alpha, \beta)a,\ a, b\in G\}$,

$T^2(X_G, G)=\{s \mid s\in C^2(X_G, G),\ s(\alpha a, \beta b, \gamma c)=b^{-1}s(\alpha, \beta, \gamma)b,\ a, b, c\in G\}$.

1. *Obstruction class for the existence of topological connection forms.* We denote by X_G the total space of the principal bundle associated to a topological G-bundle ξ over X, π the projection of X_G to X. If U is a coordinate neighborhood of ξ then, by lemma 4 of [2], $C^1(\pi^{-1}(U), G)_G$ is not an empty set and we obtain by the corollary of theorem 2 in [2]

(1)　　$C^1(\pi^{-1}(U), G)_G= T^1(\pi^{-1}(U), G)s,$

where s is a connection form of $\xi \mid U$.

On X_G, we set

$\mathcal{S}^1$: the sheaf of germs of elements of $C^1(\pi^{-1}(U), G)_G$,

$\mathcal{T}^i$: the sheaf of germs of elements of $T^i(\pi^{-1}(U), G)$, $i=1, 2$.

If we regard $\mathcal{S}^1$ and $\mathcal{T}^i$ to be sheaves on X, then we denote them by $\mathcal{S}^1_\xi, \mathcal{T}^i_\xi$ and call that $\mathcal{S}^1_\xi$ is the connection sheaf and $\delta_1\mathcal{S}^1_\xi$ is the curvature sheaf of ξ.

Since $T^i(\pi^{-1}(U), G)$ are groups, $\mathcal{T}^i$ are sheaves of groups for $i=1, 2$, but $\mathcal{S}^1$ is only a sheaf of sets. But by (1), if s_U belongs to $H^0(\pi^{-1}(U), \mathcal{S}^1)$, then $s_U s_V^{-1}$ belongs to $H^0(\pi^{-1}(U\cap V), \mathcal{T}^1)$ and we get

Lemma 1. *The class of* $\{s_U s_V^{-1}\}$ *in* $H^1(X_G, \mathcal{T}^1)$ *does not depend on the choice of* $\{s_U\}$.

Definition. The class of $\{s_U s_V^{-1}\}$ in $H^1(X_G, \mathcal{T}^1)$ is called the obstruction class for the existence of (topological) connection of ξ and denoted by $o(\xi)$.

Theorem 1. ξ *has a connection form if and only if* $o(\xi)$ *is equal to* 1 *in* $H^1(X_G, \mathcal{T}^1)$.