51. On Axiom Systems of Propositional Calculi. XV

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In his article on the protothetic [1], S. Leśniewski considered a new calculus called the equivalential calculus. This calculus is formulated as follows: Let M be an abstract set with the only undefined truth functor $\equiv$ as a primitive notion. If the set $\boldsymbol{M}=\langle M, \equiv\rangle$ satisfies the following conditions:

1 $p\equiv r.\equiv.q\equiv p: r\equiv q,$

2 $p\equiv.q\equiv r: \equiv: p\equiv q.\equiv r,$

then $\boldsymbol{M}$ is called the *equivalential calculus*.

By using the bracket, the conditions above are written in the form of

1 $((p\equiv r)\equiv(q\equiv p))\equiv(r\equiv q),$

2 $(p\equiv(q\equiv r))\equiv((p\equiv q)\equiv r).$

By a modification of Lukasiewicz symbolism, we can write these conditions as

1 $EEEprEqpErq,$

2 $EEpEqrEEpqr,$

where E is the truth functor (for example, see A. N. Prior [2]). By this symbol, the axioms of the usual equivalence relation are considered as Epp, $EEpqEqp$, and $EEpqEEqrEpr$.

In the equivalential calculus, we use the rule of usual substitution and the rule of detachment: α and $E\alpha\beta$ imply β. By these rules, S. Leśniewski proved many theses of the equivalential calculus (see [1]).

In this note, we shall use prooflines by Lukasiewicz for the proof of theses and some metatheorems given below.

Assume that the conditions 1 and 2 hold, then

2 p/r—3,

3 $EErEqrEErqr.$

1 $p/r,\ q/Erq,\ r/Eqr$ $*C3$—4,

4 $EEqrErq,$

which is a commutative law.

4 $q/EEprEqp,\ r/Erq$ $*C1$—5,

5 $EErqEEprEqp.$

1 r/q $*C4$ $q/p,\ r/q$—6,

6 $Eqq.$

Next we shall give some metatheorems on the equivalential calculus under the conditions 1 and 2. By the thesis 4, we have