50. *On Griss Algebra. I*

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About fifteen years ago, the late Professor G. F. C. Griss introduced and developed the logic of negationless intuitionistic mathematics and its mathematics (see [4], [5]). The logic is different from the two valued classical logic and the intuitionistic logic by L. E. J. Brouwer and A. Heyting (see [2], [3], and [6]). In the negationless mathematics, G. F. C. Griss rejected, in general, the notion of disjunction form. On the other hand, N. Dequoy [1] considered a projective geometry from standpoints of negationless logic and mathematics. In this note, we shall give an algebraic formulation of the negationless logic and define Griss algebra.

1. An algebraic formulation of the negationless logic. In this section, we shall take up the following formulation. Let $G=\langle X, 0, \vee, *\rangle$ be an algebra with two binary operations $\vee, *$ on a set X satisfying the following conditions:

(1) $x\vee x\leqslant x$,

(2) $x\vee y\leqslant y\vee x$,

(3) $(x\vee y)*(x\vee z)\leqslant y*z$,

(4) $x*y\leqslant (x*z)\vee(z*y)$,

(5) $x\leqslant y\vee x$,

(6) $0\leqslant x$,

(7) if $x\leqslant y$ and $y\leqslant x$, then $x=y$,

where $x\leqslant y$ is defined by $x*y=0$.

Hence $0\leqslant x$ is equivalent to $0*x=0$. Of course we can define the dual algebra of the algebra X, but we do not consider it. We first deduce some lemmas.

Axioms (1) and (5) imply

(8) $x\vee x=x$.

Hence we have

(9) $0\vee 0=0$.

From (2), we have

(10) $x\vee y=y\vee x$.

By axiom (4), $x*x\leqslant (x*(x\vee x))*((x\vee x)*x)=0$, hence

(11) $x\leqslant x$.

From axiom (4), $x*z\leqslant (x*y)\vee(y*z)$. If $x\leqslant y, y\leqslant z$, then $x\leqslant z$.

(12) $x\leqslant y, y\leqslant z$ imply $x\leqslant z$.

By (5) and (10), we have $x\leqslant 0\vee x=x\vee 0$. On the other hand, by axiom (3),