48. On Propagation of Regularity in Space-variables for the Solutions of Differential Equations with Constant Coefficients

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Introduction. Let $P(D_t, D_x)$ be a differential operator with constant coefficients for which the plane: $t=0$ is characteristic. In the note [4] K. Shinkai and the author characterized this operator P through the Gevrey class $G(\alpha)$ $(-\infty \leqq \alpha < 1)$, with respect to space-variables, in which null solutions[1)] of $Pu=0$ are able to exist.

In this note we are concerned with the converse problem: 'Is it possible to construct a null solution such that its derivative of some order has the discontinuity with respect to space-variables at some point (t_0, x_0) $(t_0 > 0)$?' Here we give a negative answer for this problem in the sense of Theorem 1. For example, the solutions of the wave equation $(\partial^2/\partial t\,\partial x)u=0$ have the form $u(t,x)=f(t)+g(x)$. Hence, if a solution of $(\partial^2/\partial t\,\partial x)u=0$ is analytic in x for negative t, then, necessarily, it is analytic in x for positive t. But, in order to generalize this phenomena, it is necessary to discuss the propagation of regularily, which has been studied by F. John [3], B. Malgrange [5], L. Hörmander [2], and J. Boman [1], with respect to only the space-variables. We shall use L^1-estimates according to J. Boman. The details will be published in the Funkcialaj Ekvacioj.

§1. Notations and preliminary lemmas. Let $(t, x)=(t, x_1, \cdots, x_\nu)$ be a point in the Euclidean $(1+\nu)$-space $R^{1+\nu}$, $\xi=(\xi_1, \cdots, \xi_\nu)$ be a point in the dual space E^ν of R^ν, and $\alpha=(\alpha_1, \cdots, \alpha_\nu)$ be a real vector whose elements are non-negative integers. We shall use notations:

$$(D_t, D_x)=(D_t, D_{x_1}, \cdots, D_{x_\nu})=(-i\partial/\partial t, -i\partial/\partial x_1, \cdots, -i\partial/\partial x_\nu),$$

$$|\alpha|=\alpha_1+\cdots+\alpha_\nu,\ \alpha!=\alpha_1!\cdots\alpha_\nu!,\ x\cdot\xi=x_1\xi_1+\cdots+x_\nu\xi_\nu,$$

$$D_x^\alpha=D_{x_1}^{\alpha_1}\cdots D_{x_\nu}^{\alpha_\nu},\ \xi^\alpha=\xi_1^{\alpha_1}\cdots\xi_\nu^{\alpha_\nu}.$$

For a function $v(x)\in C_0^\infty(R^\nu)$ we define the Fourier transform $\tilde{v}(\xi)$ by

$$\tilde{v}(\xi)=\frac{1}{\sqrt{2\pi}^{\nu}}\int_{R^\nu} e^{-ix\cdot\xi}v(x)dx$$

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1) A C^∞-solution u of $Pu=0$ is called a null solution, if $u\equiv 0$ for $t\leqq 0$ and $u\not\equiv 0$ for $t>0$.