46. *On the Completeness of the Leibnizian Modal System with a Restriction*

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§1. **Introduction.** The purpose of this paper is to show the completeness of a modal system which will be called L_0 in the following.

In my previous paper [1], in order to show an example of defence of circular definition, the following definition was given:

A statement is *analytic* if and only if it is *consistent* with every statement that expresses what is *possible*.

This definition, roughly speaking, is materially equivalent to Carnap's definition of L-truth which is suggested by Leibniz' conception that a necessary truth must hold in all possible worlds (cf. Carnap [2], p. 10).

If "analytic" is replaced by "necessary" in the above definition, this definition becomes as follows:

A statement is *necessary* if and only if it is *consistent* with every statement that expresses what is *possible*.

This reformed definition is symbolized by modal signs as follows:

$$\Box p \equiv (q)[\Diamond q \supset \Diamond (p \cdot q)],$$

where p and q are propositional variables.

Let us replace it by the following axiom-schema and rule:

Axiom-schema. $\Box \alpha \supset [\Diamond \beta \supset \Diamond (\alpha \cdot \beta)]$, where α, β are arbitrary formulas.

Rule of inference. If $\vdash \Diamond p \supset \Diamond (\alpha \cdot p)$, then $\vdash \Box \alpha$, where α is an arbitrary formula and p is a propositional variable not contained in α.

We call L (the Leibnizian modal system) the system obtained from the usual propositional calculus by adding the above axiom schema and rule and the rule of replacement of material equivalents. ($\Diamond \alpha$ is regarded as the abbreviation of $\sim \Box \sim \alpha$).

This system is easily proved to be equipollent to the system obtained from the usual propositional calculus by adding the following axiom-schema and rule and the rule of replacement:

Axiom-schema. $\Box(\alpha \supset \beta) \supset (\Box \alpha \supset \Box \beta)$.

Rule. If α is a tautology, then $\vdash \Box \alpha$.

We call L_0 the latter system with the restriction that if $\Box \alpha$ is a formula of L_0 then α does not contain $\Box$. We shall discuss the completeness of L_0 in the following sections.

§2. **Main results.** We write $\alpha, \beta, \gamma, \cdots$ for the formulas of L_0