45. Holomorphic Imbeddings of Symmetric Domains into a Symmetric Domain

By Shin-ichiro IHARA
Department of General Education, University of Tokyo

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The problem of imbedding of symmetric domains into a symmetric domain is of interest in connection with the theories of moduli and of automorphic functions. Recently, Satake has determined all holomorphic imbeddings into a Siegel space ([3], [4]). The purpose of the present note is to treat the problem by a method similar to [3], and to determine in particular all holomorphic imbeddings into the exceptional domains (*EIII*) and (*EVII*). A more detailed paper will be published elsewhere.

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1. *Definitions and Notations* (following generally to those used in [3], [4]). A semi-simple Lie algebra $\mathfrak{g}$ over $\boldsymbol{R}$ is called of *hermitian type* if a maximal compact subalgebra of each non-compact simple factor has non-trivial center. Let $G=\mathrm{Int}(\mathfrak{g})$ be the group of all inner automorphisms of $\mathfrak{g}$, K a subgroup of G corresponding to a maximal compact subalgebra $\mathfrak{k}$; let further $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the Cartan decomposition (corresponding to $\mathfrak{k}$). Then the symmetric space $D=G/K$ has a G-invariant complex structure and thus becomes a symmetric domain, and for each of such complex structures, there exists a uniquely determined H_0 in the center of $\mathfrak{k}$ such that $ad(H_0)$ induces on $\mathfrak{p}$, as the tangent space to D at the origin K, the given complex structure. Such an element H_0 is called an *H-element* of $\mathfrak{g}$ (relative to the Cartan decomposition). If $\mathfrak{g}$ is simple, D is irreducible and H-element is uniquely determined up to the sign $\pm$. The usual symbols $(I)_{p,q}$, $(II)_p$, $(III)_p$, $(IV)_p$, $(EIII)$, and $(EVII)$ for irreducible symmetric domain will be also used to denote the corresponding Lie algebras. By $\mathfrak{g}_C, \cdots$, we express the complexifications of $\mathfrak{g}, \cdots$.

All the fundamental properties of symmetric domains used in this paper will be found in [2].

2. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ and $\mathfrak{g}'=\mathfrak{k}'+\mathfrak{p}'$ be semi-simple Lie algebras of hermitian type, and H_0, H_0' be H-elements of $\mathfrak{g}$, $\mathfrak{g}'$ respectively. We consider the problem in the following form (see [3]): *For given semi-simple Lie algebras $\mathfrak{g}$ and $\mathfrak{g}'$ of hermitian type, determine all equivalence-classes of homomorphisms ρ of $\mathfrak{g}$ into $\mathfrak{g}'$ satisfying*