106. *A Generalization of the Cauchy Filter and the Completion*

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In this paper, to take away the notion of covering system and we consider about the completion theory of topological space with a set consisting of some filters instead of Cauchy filters concerning covering system.

Thus, we get a generalization of author's paper [5], but using method is not different almost at all.

By this generalization, Alexandroff one point compactification is included, as a special case, in the completion.

A family $\mathfrak{f}$ consisting of subsets of X is a *filter base* in X if for every $A, B \in \mathfrak{f}$, $C \subseteq A \cap B$ for some $C \in \mathfrak{f}$ and $\phi \notin \mathfrak{f}$.

A *filter* $\mathfrak{f}$ in X is a filter base in X such that if $A \supseteq B$ and $B \in \mathfrak{f}$ then $A \in \mathfrak{f}$.

For every filter base $\mathfrak{f}$ in X, the family $\{A \mid X \supseteq A \supseteq B, B \in \mathfrak{f}\}$ is a filter in X, that is said to be *generated* by $\mathfrak{f}$.

If $X^* \supseteq X$ then a filter $\mathfrak{f}$ in X is a filter base in X^* and generates a filter in X^*. Denote it by $\mathfrak{f}^*$.

In a topological space X, let's denote by $\mathfrak{N}(x)$, the neighborhood system of $x \in X$, and by $\mathfrak{G}(X)$, the family of all open sets of X.

A filter base $\mathfrak{f}$ in a topological space X *converges* to x in X if and only if the filter generated by $\mathfrak{f}$ contains the neighborhood system $\mathfrak{N}(x)$ of x.

For a filter base $\mathfrak{f}$ in a topological space X, $\{G \mid G \in \mathfrak{G}(X), G \supseteq A, A \in \mathfrak{f}\}$ is a filter base, so generates a filter, we will denote it by $\mathfrak{f}^\pi$. Thus $\mathfrak{f}^\pi$ converges to x if and only if $\mathfrak{f}$ converges to x.

We consider a topological space X, with a set M consisting of some filters that satisfies the following three conditions

M1) if $\mathfrak{f} \in M$ and $\mathfrak{g} \supseteq \mathfrak{f}$ then $\mathfrak{g} \in M$,

M2) if $\mathfrak{f} \in M$ then $\mathfrak{f}^\pi \in M$,

M3) for all point x of X, $\mathfrak{N}(x) \in M$.

Let's denote such a topological space X, by $(X; M)$. In $(X; M)$, if $\mathfrak{f} \in M$ converges to no point, then $\mathfrak{f}$ is a *leg*. If $(X; M)$ has no leg, $(X; M)$ is *complete*.

A *completion* $(X^*; M^*)$ of a space $(X; M)$ is such a space that

C1) $X \subseteq X^*$,