

104. On Variants of Axiom Systems of Propositional Calculus. II

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(Comm. by Kinjirô KUNUGI, M.J.A., May 12, 1966)

In this paper, we shall show that any axiom system containing the *BCK*-system of propositional calculus may be changed into a new system which has axioms less than the number of the original axioms. Following certain 'combinatory logicians', we put *B* for $CCqrCCpqCpr$, *C* for $CCpCqrCqCpr$, and *K* for $CpCqp$. This system was given by C. A. Meredith. Further Prof. K. Iséki has given the algebraic formulation of the *BCK*-system (see, [5]). For the notations and two rules of inference, see [3].

The axioms of the *BCK*-system are given by the following:

- 1' $CCqrCCpqCpr$,
- 2' $CCpCqrCqCpr$,
- 3' $CpCqp$.

It is well known that these axioms imply (see, [1]),

- 4' $CCpqCCqrCpr$,
- 5' $CpCCpqq$.

Theorem 1. *If F and G are the formulas in the *BCK*-system, then $CpCqp$ and $CCGCFvCwv$ imply F and G , where F and G do not contain v and w .*

Proof. We put

- 1 $CpCqp$,
- 2 $CCGCFvCwv$.
2 $v/w, w/CpCqp *C1 p/G, q/F—C1—3$,
- 3 G .
1 $p/CpCqp *C1—4$,
- 4 $CqCpCqp$.
2 $v/CGF, w/G *C4 p/F, q/G—C3—C3—5$,
- 5 F .

Theorem 2. *If F and G are the formulas in the *BCK*-system, then $CCGCFvCwv$ is a formula in this system.*

Proof. 5' $p/F, q/v *CF—1$,

- 1 $CCFvv$.
1' $p/G, q/CFv, r/v *C1—2$,
- 2 $CCGCFvCGv$.
2' $p/CGCFv, q/G, r/v *C2—CG—3$,
- 3 $CCGCFvv$.