# 102. *On Characterizations of I-Algebra. I*

By Yasuo SETÔ and Shôtarô TANAKA

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In this paper, we shall show that *an axiomatic system of implicational calculus given by C. A. Meredith is equivalent to Tarski-Bernays' axiom system* using an algebraic formulation.

In his paper [2], Prof. K. Iséki has proved that Tarski-Bernays' axiom system implies Meredith's system and other systems. Further Prof. K. Iséki refers Tarski-Bernays' axiom system as $I$-algebra. We shall prove that Meredith's alternative 4-axiom set implies Tarski-Bernays' system. We shall carry out this proof algebraically.

Let $\langle X, 0, *\rangle$ be an abstract algebra. For the notion of this algebra and notations, see [1]. The alternative 4-axiom set is given as the following 1–4, $D1$–$D3$.

1 $y*(y*x)\leqslant x$,

2 $(z*x)*(z*y)\leqslant y*x$,

3 $y*x\leqslant (y*x)*x$,

4 $x*(x*y)\leqslant y*(y*x)$,

$D1$ $x\leqslant y$ means $x*y=0$,

$D2$ $0\leqslant x$,

$D3$ $x\leqslant y$, $y\leqslant x$ imply $x=y$.

In 2, put $y*(y*x)$ for $y$, then we have

$(z*x)*(z*(y*(y*x)))\leqslant (y*(y*x))*x$.

By 1 the right side of the above is equal to 0. Hence, by $D1$, $D2$, and $D3$, we have

5 $z*x\leqslant z*(y*(y*x))$.

If we put $x=(z*y)*x$, $y=(z*x)*(z*(z*y))$, $z=(z*x)*y$ in 2, then we have

$$(((z*x)*y)*((z*y)*x))*(((z*x)*y)*((z*x)*(z*(z*y))))$$
$$\leqslant ((z*x)*(z*(z*y)))*((z*y)*x).$$

We see the right side is equal to 0, putting $y=z*y$ in 2. At the same time, we see the second term of the left side is equal to 0, putting $x=y$, $y=z$, $z=z*x$ in 5. Hence we have

6 $(z*x)*y\leqslant (z*y)*x$.

In 2, put $y=y*(y*x)$, $z=x*(x*y)$, and apply 1, 2 to it, we have

7 $(x*(x*y))\leqslant x$.

In 6, put $y=x*y$, $z=x$, then we have $(x*x)*(x*y)\leqslant (x*(x*y))*x$. By 7, the right side is equal to 0. Hence we have

8 $x*x\leqslant x*y$.