99. *Obstructions to Locally Flat Embeddings of Bounded Combinatorial Manifolds*

By Masako UJIHARA
Waseda University, Tokyo

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In the paper [1], H. Noguchi showed that for any proper $(p+1)$-flat embedding $f: M\rightarrow W$, where M is an oriented closed n-manifold and W is an oriented $(n+2)$-manifold without boundary, the p-homology class Ω_f of M, called the Whitehead class of f, is defined, and if $\Omega_f=0$, the embedding f can be arbitrarily approximated by a p-flat embedding $g: M\rightarrow W$, $0\leq p\leq n-2$.

We will extend this for bounded manifolds M and W as follows.

Let M be a compact oriented n-manifold with non-vacuous boundary ∂M, and W be an oriented $(n+2)$-manifold with non-vacuous boundary ∂W. Let $f: M\rightarrow W$ be a proper embedding; that is to say, $f(\operatorname{Int} M)\subset \operatorname{Int} W$ and $f(\partial M)\subset\partial W$. Then, by § 4 of [1], f is $(n-1)$-flat. Hence it is assumed that f is a $(p+1)$-flat embedding, $0\leq p\leq n-2$.

Next we define the p-homology class $\Omega_f\in H_p(M, \partial M; G^{n-p-1})$ of M mod ∂M, called the Whitehead class of the embedding f, where G^{n-p-1} is the knot cobordism group of dimension $n-p-1$. In fact by Theorem 3 of [2] (see § 1 of [1]), the class Ω_f is invariant under the iso-neighboring relation of proper embeddings of M in W.

The main result of the paper is as follows.

Theorem. *If the Whitehead class Ω_f of f is the identity, f can be arbitrarily approximated by a p-flat embedding.*

If C is an n-cell, then $H_p(C, \partial C; G^{n-p-1})=0$ for $0\leq p\leq n-2$, and we have the following.

Corollary 1. *Let C, D be n-, $(n+2)$-cells and $f: C\rightarrow D$ be a proper embedding. Then f is arbitrarily approximated by a locally flat embedding.*

Since $H_0(M, \partial M; G^{n-1})=0$ for each manifold M with non-vacuous boundary ∂M, we have the following.

Corollary 2. *Any 1-flat proper embedding $f: M\rightarrow W$ can be arbitrarily approximated by a locally flat embedding.*

From now on it will be assumed that the embedding $f: M\rightarrow W$ is $(p+1)$-flat.

Notation. Let $\varphi: K\rightarrow L$ be a triangulation of f. Then ∂K means a subcomplex of K covering ∂M, and $\operatorname{Int} K$ means the set of simplexes $K-\partial K$. Let $\triangle$ be an oriented r-simplex of ∂K. Then $\nabla_\partial(\square_\partial)$ is an