128. Existence and Uniqueness of Extensions of Volumes and the Operation of Completion of a Volume. I*)

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Introduction. Let R, Y be the space of reals and a Banach space respectively. The norm of elements in these spaces will be denoted by $|\ \ |$.

A nonempty family of sets V of an abstract space X will be called a *pre-ring* if for any two sets $A_1, A_2 \in V$ we have $A_1 \cap A_2 \in V$, and there exist disjoint sets $B_1, \cdots, B_k \in V$ such that $A_1 \backslash A_2 = B_1 \cup \cdots \cup B_k$.

A non-negative finite-valued function v on the pre-ring V will be called a *volume* if for every countable family of disjoint sets $A_t \in V (t \in T)$ such that $A = \bigcup_T A_t \in V$ we have $v(A) = \sum_T v(A_t)$.

In [1] has been presented a direct construction of the space $L(v, Y)$ of *Lebesgue-Bochner summable functions* and has been developed the theory of an integral of the form $\int u(f, d\mu)$. In the case when the bilinear form is given by $u(y, z) = zy$ for $y \in Y$, $z \in R$ and $\mu = v$ the above integral coincides with the classical Lebesgue-Bochner integral $\int f dv$.

All basic theorems concerning the algebraical and topological structures of the space $L(v, Y)$ have been proven without developing the theory of measure or the theory of measurable functions.

Basing the theory of integration on set functions defined on pre-rings it was possible in [2], [3] to develop the theory of *multilinear vectorial integration* and define *integral representations of multilinear continuous operators* on the space of Lebesgue-Bochner summable functions. It also permitted us to give new constructions of *Fubini's theorem* and to find its farther generalizations [4].

The theory of *Lebesgue-Bochner measurable functions* corresponding to the approach developed in [1] has been presented in [5]. The theory of *measure* has been obtained as a by-product of the theory of integration.

These results permitted us to simplify the theory of integration

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