166. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XXIII

By Sakuji INOUE
Faculty of Science, Kumamoto University

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1966)

Let D_j ($j=1$ to n), $\{\lambda_\nu\}_{\nu=1,2,3,\dots}$, N_j ($j=1$ to n), $f_{1\alpha}, f_{2\alpha}, f'_{1\alpha}, f'_{2\alpha}, g_{j\beta}$, and $g'_{j\beta}$ be the same notations as those defined in Part XIII [cf. Proc. Japan Acad., Vol. 40, pp. 492–497 (1964)], and $R(\lambda)$ an integral function. Throughout this paper we deal with a resolvent function $\tilde{U}(\lambda)$ concerning the bounded normal operators N_j such that

$$\begin{aligned}\tilde{U}(\lambda)&=R(\lambda)+\sum_{\alpha=1}^{\infty}((\lambda I-N_1)^{-\alpha}(f_{1\alpha}+f_{2\alpha}),\, f'_{1\alpha}+f'_{2\alpha})+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}((\lambda I-N_j)^{-\beta}g_{j\beta},\, g'_{j\beta})\\&=R(\lambda)+\sum_{\alpha=1}^{\infty}\sum_{\nu=1}^{\infty}c_\alpha^{(\nu)}(\lambda-\lambda_\nu)^{-\alpha}+\sum_{\alpha=1}^{\infty}\int_{[\overline{\{\lambda_\nu\}}-\{\lambda_\nu\}]\cup D_1}(\lambda-\zeta)^{-\alpha}d(K^{(1)}(\zeta)f_{2\alpha},\, f'_{2\alpha})\\&\quad+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}\int_{D_j}(\lambda-\zeta)^{-\beta}d(K^{(j)}(\zeta)g_{j\beta},\, g'_{j\beta})\end{aligned}$$

where $\{K^{(j)}(\zeta)\}$ denotes the complex spectral family of N_j for each value of $j=1, 2, 3, \cdots, n$, on the assumptions that

$$\sum_{\alpha=1}^{\infty}\sum_{\nu=1}^{\infty}|c_\alpha^{(\nu)}(\lambda-\lambda_\nu)^{-\alpha}|<\infty \qquad (\lambda\notin\overline{\{\lambda_\nu\}})$$

and

$$\sum_{\alpha=1}^{\infty}\left|\int_{[\overline{\{\lambda_\nu\}}-\{\lambda_\nu\}]\cup D_1}(\lambda-\zeta)^{-\alpha}d(K^{(1)}(\zeta)f_{2\alpha},\, f'_{2\alpha})\right|<\infty \qquad (\lambda\notin\overline{\{\lambda_\nu\}}\cup D_1).$$

In fact, as will be seen from the method used to show that there exist uncountably many pairs of $f_{1\alpha}$ and $f'_{1\alpha}$ such that the former inequality holds [cf. Proc. Japan Acad., Vol. 42, pp. 583–588 (1966)], we can find uncountably many pairs of $f_{2\alpha}$ and $f'_{2\alpha}$ such that the latter inequality holds.

Theorem 64. Let $\tilde{U}(\lambda)$ be the function defined above, and let $\overline{\{\lambda_\nu\}}\cup\left[\bigcup_{j=1}^{n}D_j\right]$ be contained in the disc $\bar{\mathfrak{D}}_\sigma\{\lambda: |\lambda|\leqq\sigma\}$. Then $\tilde{U}(\lambda)$ is expansible on any domain $\Delta_\rho\{\lambda: \rho<|\lambda|<\infty\}$ with $\sigma<\rho<\infty$ in the form

$$\tilde{U}\left(\frac{\rho}{\kappa}e^{i\theta}\right)=\frac{1}{2}a_0+\frac{1}{2}\sum_{p=1}^{\infty}(a_p-ib_p)\left(\frac{e^{i\theta}}{\kappa}\right)^p+\frac{1}{2}\sum_{p=1}^{\infty}(a_p+ib_p)\left(\frac{\kappa}{e^{i\theta}}\right)^p \quad (0<\kappa<1)$$

where

$$\left.\begin{aligned}a_p&=\frac{1}{\pi}\int_0^{2\pi}\tilde{U}(\rho e^{it})\cos ptdt\\ b_p&=\frac{1}{\pi}\int_0^{2\pi}\tilde{U}(\rho e^{it})\sin ptdt\end{aligned}\right\} \qquad (p=0, 1, 2, \cdots) \tag{52}$$