161. *Transformation of Branching Markov Processes*

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General theory for transformations of Markov processes by multiplicative functionals is well known (cf. Dynkin [1], Meyer [6], Kunita and T. Watanabe [5], and Ito and S. Watanabe [4]). Let X_t be a given Markov process and M_t be a multiplicative functional of X_t satisfying $E_x[M_t]\leqq 1$, then M_t-subprocess X_t^M of X_t is a Markov process with the transition probability $P^M(t, x, dy)=E_x[M_t\chi_{dy}(X_t)]$. When X_t is a Branching Markov process, the transformed process X_t^M happens to be not a branching Markov process. For example, if $M_t=\exp(-t)$, X_t^M is not branching Markov process. In this paper, we shall investigate necessary and sufficient conditions for M_t under which X_t^M becomes a branching Markov process, and give some examples of transformation.

1. Multiplicative functionals of branching type. Let $X_t(w)$ be a branching Markov process taking values in $\boldsymbol{S}=\bigcup_{n=0}^{\infty} S^n$, where $S^0=\{\partial\}$ and $S^\infty=\{\Delta\}$. For convenience, we assume in this paper that the process is defined on the path space W of right continuous paths and that $X_t(w)$ represents the position taken by a path $w\in W$ at time $t\geqq 0$. Precise definition of branching Markov process has been given in [2]. We shall use the terminology and the notation adopted in [2] and [3].

Let $W^{(n)}$ be the n-fold product of W and put $\tilde{W}=\bigcup_{n=0}^{\infty} W^{(n)}$, where $W^{(0)}=\{w_\partial\}$ $W^{(\infty)}=\{w_\Delta\}$.[1] We shall define a mapping ϕ of $\tilde{W}$ to W by

$$X_t(\phi\tilde{w})=(\phi\tilde{w})(t)=\gamma\{X_t(w^1), X_t(w^2), \cdots, X_t(w^n)\}, \quad t\geqq 0, \tag{1.1}$$

when $\tilde{w}=(w^1, w^2, \cdots, w^n)\in W^{(n)}$, $w^j\in W$, $j=1, 2, \cdots, n$. Where γ is the mapping from $\bigcup_{n=1}^{\infty} \boldsymbol{S}^{(n)}$ to $\boldsymbol{S}$.[2]

Definition 1.1. Let M_t be an $\mathcal{N}_t$-multiplicative functional[3] of a

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1) $X_t(w_\partial)=\partial$, for all $t\geqq 0$. $X_t(w_\Delta)=\Delta$, for all $t\geqq 0$.

2) $\boldsymbol{S}^{(n)}$ is the n-fold product of $\boldsymbol{S}$. The definition of the mapping γ is given in [2].

3) Let $X=\{W, X_t, \mathcal{B}_t, P_x, x\in S\}$ be a right continuous Markov process. In this

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