221. On Branching Semi-Groups. I

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In the previous papers we have given a definition of branching Markov processes (abbreviated as B.M.P.) [1], discussed some fundamental equations of B.M.P. [2], and constructed B.M.P. in a probabilistic way [3], [4]. This paper is a continuation of these papers and is devoted to an analytic construction of B.M.P. We shall treat this problem, however, in a little wider setup which may permit us to deal with the not necessarily positive branching semi-groups. (c.f. [5]).

1. Definition of branching semi-groups. Let S be a compact Hausdorff space with countable base, S^n be the $\boldsymbol{n}$-fold symmetric product of S ($S^0=\{\partial\}$, an isolated point), and $\boldsymbol{S}=\bigcup_{n=0}^{\infty}S^n\cup\{\Delta\}$ be the one-point compactification of $\bigcup_{n=0}^{\infty}S^n$.[1] We denote by $\boldsymbol{C}(\boldsymbol{S})$ (resp. $\boldsymbol{C}(S)$ and $\boldsymbol{C}(S^n)$) the space of bounded continuous functions on $\boldsymbol{S}$ (resp. on S and S^n). $\boldsymbol{B}(\boldsymbol{S})$ is the space of bounded Borel measurable functions on $\boldsymbol{S}$. $\boldsymbol{C}_0(\boldsymbol{S})$ (resp. $\boldsymbol{B}_0(\boldsymbol{S})$) is the subspace of $\boldsymbol{C}(\boldsymbol{S})$ (resp. $\boldsymbol{B}(\boldsymbol{S})$) the elements of which vanish at infinity Δ.

Definition 1.1. A contraction[2] semi-group $\{T_t; t\geqq 0\}$ of linear operators on $\boldsymbol{C}(\boldsymbol{S})$ (or $\boldsymbol{B}(\boldsymbol{S})$) is said to be a *branching semi-group* (or *of branching property*), if it satisfies

$$T_t\hat{f}(\boldsymbol{x})=\widehat{(T_t\hat{f}|_S)}(\boldsymbol{x}),\qquad \boldsymbol{x}\in\boldsymbol{S},^{3)} \tag{1.1}$$

for any $f\in\bar{C}^*(S)$ (or $\bar{B}^*(S)$).[4]

Remark. Let $\boldsymbol{B}$ be a Banach space or Hilbert space, $\boldsymbol{B}^n=\boldsymbol{B}\otimes\boldsymbol{B}\otimes\cdots\otimes\boldsymbol{B}$ be the n-fold symmetric direct product of $\boldsymbol{B}$, and $\mathscr{B}=\sum_{n=0}^{\infty}\oplus\boldsymbol{B}^n$ ($B^0=\{\text{constants}\}$) be the direct sum of $\boldsymbol{B}^n$. Then the notion of branching semi-groups can be extended to a semi-group of linear operators on $\mathscr{B}$.

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1) For precise definition of $\boldsymbol{S}$, we refer to [1].

2) i.e. $\|T_t\|\leqq 1$. We do not assume positivity of T_t.

3) For $f\in\bar{\boldsymbol{B}}^*(S)$, we put $\hat{f}(\boldsymbol{x})=\prod_{j=1}^{n}f(x_j)$ if $x\in S^n$, $=0$ if $\boldsymbol{x}=\Delta$, and $=1$ if $\boldsymbol{x}=\partial$.

4) $\boldsymbol{C}^*(S)$ ($\boldsymbol{B}^*(S)$)$=\{f; f$ is bounded continuous (resp. Borel measurable) with $\|f\|<1\}$. $\bar{\boldsymbol{C}}^*(S)$ ($\bar{\boldsymbol{B}}^*(S)$) is the uniform closure of $\boldsymbol{C}^*(S)$ ($\boldsymbol{B}^*(S)$).