220. A Note on the Inductive Dimension of Product Spaces

By Yûkiti KATUTA
Ehime University

(Comm. by Kinjirô KUNUGI, M.J.A., Nov. 12, 1966)

The *large inductive dimension* of a space[1] X, Ind X, is defined inductively as follows. If X is the empty set, Ind $X=-1$. For $n=0, 1, 2, \cdots$, Ind $X\leqq n$ means that for any pair of a closed set F and an open set G with $F\subset G$ there exists an open set U such that $F\subset U\subset G$, Ind $(\bar{U}-U)\leqq n-1$. Ind $X=n$ means that Ind $X\leqq n$ and the statement Ind $X\leqq n-1$ is false. Ind $X=\infty$ means that the statement Ind $X\leqq n$ is false for any n.

In [11], K. Nagami proved that the inequality

$$\text{(1)} \qquad \operatorname{Ind}(X\times Y)\leqq \operatorname{Ind} X+\operatorname{Ind} Y$$

holds for the case where X is a perfectly normal paracompact space and Y is a metric space. Then N. Kimura [5] generalized the above result of Nagami by proving that the inequality (1) holds for the case where Y is a metric space and $X\times Y$ is a countably paracompact, totally normal space. Here the notion of totally normal spaces was defined by C. H. Dowker [3] and he proved that the subset theorem and the sum theorem hold for the large inductive dimension of totally normal spaces.

On the other hand, as for the covering dimension of product spaces, K. Morita [9] proved that the inequality

$$\dim(X\times Y)\leqq \dim X+\dim Y$$

holds for the following three cases: (a) $X\times Y$ is an S-space, where a space R is said to be an *S-space* if every open covering of R has a star-finite open refinement, (b) X is a paracompact space and Y is a locally compact paracompact space and (c) X is a countably paracompact normal space and Y is a locally compact metric space.

In this note we shall prove that the above inequality (1) holds for the following two cases:

I. $X\times Y$ is a totally normal S-space.

II. X is a paracompact space, Y is a locally compact paracompact space and $X\times Y$ is a totally normal space.

Our proof for Case I is based on the fact that if R is a totally normal S-space we have Ind $R=$ ind R. Here the *small inductive dimenion* of a space X, ind X, is defined inductively as follows. If X is the empty set, ind $X=-1$. For $n=0, 1, 2, \cdots$, ind $X\leqq n$

1) Throughout this note a *space* means a Hausdorff space.