18. *On the Absolute Logarithmic Summability of the Allied Series of a Fourier Series*

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1. Introduction. 1.1. Definition.*) *Let $\lambda=\lambda(w)$ be continuous, differentiable and monotone increasing in $(0,\infty)$, and let it tend to infinity as $w\to\infty$. For a given series $\sum_1^\infty a_n$, we put*

$$C_r(w)=\sum_{n\leqslant w}\{\lambda(w)-\lambda(n)\}^r a_n \qquad (r\geqslant 0).$$

Then the series $\sum_1^\infty a_n$ is called to be summable $|R,\lambda,r|$ $(r\geqslant 0)$, if

$$\int_A^\infty \left| d\left[\frac{C_r(w)}{(w)^r}\right]\right|<\infty \tag{1.1.1}$$

for a positive number A.

For $r>0$, and non-integral w, we have

$$\frac{d}{dw}\left[\frac{C_r(w)}{\{\lambda(w)\}^r}\right]=\frac{r\lambda'(w)}{\{\lambda(w)\}^{1+r}}\sum_{n\leqslant w}\{\lambda(w)-\lambda(n)\}^{r-1}\lambda(n)a_n.$$

Hence $\sum_1^\infty a_n$ is summable $|R,\lambda,r|$ $(r>0)$, if and only if

$$\int_A^\infty\left|\frac{r\lambda'(w)}{\{\lambda(w)\}^{1+r}}\sum_{n\leqslant w}\{\lambda(w)-\lambda(n)\}^{r-1}\lambda(n)a_n\right|dw<\infty. \tag{1.1.2}$$

1.2. We suppose that $f(t)$ is integrable in the Lebesgue sense in the interval $(-\pi,\pi)$, and is periodic with period 2π, so that

$$f(t)\sim\tfrac{1}{2}a_0+\sum_1^\infty(a_n\cos nt+b_n\sin nt)=\tfrac{1}{2}a_0+\sum_1^\infty A_n(t). \tag{1.2.1}$$

Then the allied series is

$$\sum_1^\infty(b_n\cos nt-a_n\sin nt)=\sum_1^\infty B_n(t). \tag{1.2.2}$$

We write

$$\psi(t)=\tfrac{1}{2}\{f(x+t)-f(x-t)\},\quad \theta(t)=\int_t^\pi\frac{\psi(u)}{u}du. \tag{1.2.3}$$

The object of the present paper is to prove the following

Theorem. *If $t^{-1}|\theta(t)|\log\frac{2\pi}{t}\in L(0,\pi)$, then* (1.2.2) *is summable $|R,\log w,2|$ at $t=x$.*

This theorem was conjectured by N. Basu in a stronger form.

2. Proof of the Theorem. 2.1. We write

$$g(w,t)=\sum_{n<w}\log n\left(\log\frac{w}{n}\right)\sin nt, \tag{2.1.1}$$

*) Mohanty (1).