39. A Note on Jacobi Fields of δ-Pinched Riemannian Manifolds

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In [1] M. Berger stated a theorem relating to Jacobi fields of a complete δ-pinched Riemannian manifold which is an extension of Rauch's metric comparison theorem [3]. This theorem is equivalent to the following

Proposition A. *Let M be a complete Riemannian manifold whose sectional curvature K satisfies the inequality*

(1) $$0<\delta\leqq K\leqq 1$$

and X be any Jacobi field along a geodesic $x=\gamma(s)$ parameterized with arc length s such that

(2) $$\langle X(0), \gamma'(0)\rangle=0, X'(0)=0, \qquad \| X(0) \| =1,$$

then

(3) $$\| X(s) \| \leqq \cos \sqrt{\delta} s \quad for \quad 0\leqq s\leqq \pi/2.$$

Berger's proof of Theorem 1 in [1] is due to the principle of variation analogous to Rauch's method in [2] but it is not clear whether this theorem is true or not by his exposition only. Since it can be shown that Proposition A is true in the case dim $M=2$, it may be considered that it is a conjecture in the case dim $M>2$. In this note, the author will show that this proposition holds for a locally symmetric Riemannian manifold.

Let M be an n-dimensional Riemannian manifold and X a Jacobi field along a geodesic $x=\gamma(s)$ parameterized with arc length s. Then, X satisfies the following equation

(4) $$\frac{D^2X}{ds^2}+R\left(\frac{d\gamma}{ds}, X\frac{d\gamma}{ds}\right)=0,$$

where D denotes the covariant differentiation of M, $\frac{d\gamma}{ds}$ the tangent vector of the curve $x=\gamma(s)$ and R the curvature tensor field of M.

Now, let k be a positive constant and suppose that

$$X(s)\neq 0, y(s)\equiv \cos ks\neq 0$$

in an interval $0\leqq s<s_0$. In this interval, we put

(5) $$\varphi=\frac{\langle X, y'X-yX'\rangle}{\| X \|},$$

where $\langle , \rangle$ denotes the inner product in M and $X'=\frac{DX}{ds}$. If we have