70. *On Regularity of Solutions of Abstract Differential Equations in Banach Space*

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(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1967)

The present paper is concerned with the estimates for the successive derivatives of solutions of abstract differential equations of parabolic type in a Banach space X:

$$du(t)/dt+A(t)u(t)=f(t), \qquad 0<t\leqq T. \tag{1}$$

The main result is briefly stated as follows: if $A(t)$ and $f(t)$ belong to a Gevrey's class as functions of t, then so does the solution of (1). This is an answer to the problem proposed in p. 388 of [3].

Let $\{M_k\}$ be a sequence of positive numbers which has the properties (1.1), $\cdots$, (1.7) in p. 366 of [4]. In what follows we will not confine ourselves to non quasi-analytic cases since we will not work only in the spaces such as D_{+,M_k} (cf. [3]).

Assumptions. (i) For each $t\in[0, T]$, $A(t)$ is a densely defined linear closed operator in X. The resolvent set of $A(t)$ contains a fixed closed sector $\sum=\{\lambda: \theta\leqq \arg\lambda\leqq 2\pi-\theta\}$, $0<\theta<\pi/2$.

(ii) $A(t)^{-1}$, which is a bounded operator according to the preceding assumption, is infinitely differentiable in t.

(iii) There exist constants K_0 and K such that for any $\lambda\in\sum$, $t\in[0, T]$ and non-negative integer n

$$\|(\partial/\partial t)^n(\lambda-A(t))^{-1}\|\leqq K_0K^nM_n/|\lambda|.$$

It can be shown with the aid of S. Agmon's result on general elliptic boundary value problems ([1]) that the assumptions above are satisfied for the initial-boundary value problems of parabolic differential equations under appropriate conditions on the coefficients.

In view of Theorem 3.1 of [2] the evolution operator $U(t, s)$ can be constructed as follows:

$$U(t, s)=\exp(-(t-s)A(t))+W(t, s),$$
$$W(t, s)=\int_s^t \exp(-(t-\tau)A(t))R(\tau, s)d\tau,$$
$$R(t, s)=\textstyle\sum_{m=1}^{\infty} R_m(t, s),$$
$$R_1(t, s)=-(\partial/\partial t+\partial/\partial s)\exp(-(t-s)A(t)),$$
$$R_m(t, s)=\int_s^t R_1(t, \tau)R_{m-1}(\tau, s)d\tau, \qquad m=2, 3, \cdots.$$

$R(t, s)$ is the solution of the integral equation

$$R(t, s)=R_1(t, s)+\int_s^t R_1(t, \tau)R(\tau, s)d\tau. \tag{2}$$