60. *Unions of Strongly Paracompact Spaces*

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As is known, a space that is a union of two closed strongly paracompact subspaces need not be strongly paracompact (see [7]). In our previous note ([8]; Theorem 2), we have proved the following theorem:

Let X be a regular T_1-space with a locally finite and star countable closed covering $\{F_\alpha \mid \alpha \in A\}$ such that $Fr(F_\alpha)$ has the locally Lindelöf property and F_α is strongly paracompact for any $\alpha \in A$, then X is itself strongly paracompact.

The main purpose of this note is to omit the hypothesis of the star countability from the above theorem. As the terminologies and notations in this note, refer to our previous note [8].

§1. The main theorem. In this section, we shall give the proof of the following theorem:

Theorem 1. *Let $\mathfrak{F}'=\{F'_\alpha \mid \alpha \in A\}$ be a locally finite closed covering of a regular T_1-space X such that $Fr(F'_\alpha)$*[1] *has the locally Lindelöf property for any $\alpha \in A$. Then a necessary and sufficient condition that X be strongly paracompact is that F'_α is strongly paracompact for any $\alpha \in A$.*

Proof. The necessity is obvious and so we shall prove the sufficiency. It is obvious that X is paracompact ([2]; Theorem 1). Now let A be well ordered and $F_0=F'_0$, $F_\alpha=\overline{F'_\alpha-\bigcup_{\beta<\alpha} F'_\beta}$ for each $\alpha>0$ and $Q=\bigcup_{\alpha\neq\beta}(F_\alpha\cap F_\beta)$. Then, similarly as the proof of ([8]; Theorem 1), we can show that $\mathfrak{F}=\{F_\alpha \mid \alpha \in A\}$ is a locally finite closed covering of X and Q is paracompact closed subspace with the locally Lindelöf property (see [8]; Lemma 1 and Lemma 2). Therefore we can get the discrete covering $\mathfrak{G}=\{G_\lambda \mid \lambda \in \Lambda\}$ of Q such that G_λ has the Lindelöf property for any $\lambda \in \Lambda$ by ([4]; Lemma 2.5).

In order to show the strong paracompactness of X, let $\mathfrak{W}$ be an arbitrary open covering of X, and then we shall show that $\mathfrak{W}$ has a star countable open covering of X as a refinement. At first, by the facts that $\mathfrak{G}$ is a discrete collection in the closed subspace Q and $\mathfrak{F}$ is locally finite, there exists a locally finite open covering

1) $Fr(F)$ will denote the boundary of F in X, i.e. $Fr(F)=\overline{F}\cap(\overline{X-F})$.