161. *A Characterization of Lukasiewiczian Algebra. II*

By Corneliu O. SICOE
Calculus Centre of the Bucharest University

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In [4], I introduced according to algebraic technique used by Prof. K. Iséki [1], the notion of L-algebra and I showed that a L-algebra is a three-valued Lukasiewicz algebra. In this note I shall show that a three-valued Lukasiewicz algebra is a L-algebra, hence the notion of L-algebra is equivalent with the notion of three-valued Lukasiewicz algebra.

A three-valued Lukasiewicz algebra is [3] a system $\langle A, 1, \sim, \mu, \cap, \cup \rangle$ such that the following axioms are verified:

A1) $x \cup 1 = 1$,

A2) $x \cap (x \cup y) = x$,

A3) $x \cap (y \cup z) = (z \cap x) \cup (y \cap x)$,

A4) $x = \sim\sim x$,

A5) $\sim(x \cap y) = \sim x \cup \sim y$,

A6) $\sim x \cup \mu x = 1$,

A7) $x \cap \sim x = \sim x \cap \mu x$,

A8) $\mu(x \cap y) = \mu x \cap \mu y$.

In a three-valued Lukasiewicz algebra hold the followings:

1) $\mu\mu x = \mu x$,

2) $\sim\mu\sim\mu x = \mu x$,

3) $\mu(x \cup y) = \mu x \cup y\mu$,

4) $x \cap \sim x \leq y \cup \sim y$,

5) $x \cap \mu\sim x = x \cap \sim x$,

6) $x \cap \sim\mu x = 0$,[1]

7) $\sim x \cap \sim\mu\sim x = 0$,

8) $\mu x \cap \sim\mu x = 0$,

9) $\mu\sim x \cap \sim\mu\sim x = 0$,

10) $\sim\mu x \cap \sim\mu\sim x = 0$,

11) $\sim\mu\sim x \leq x \leq \mu x$,

12) $x \cap y = 0 \rightleftarrows \mu y \leq \sim\mu x$,

13) $\mu x = \mu y$ and $\sim\mu\sim x = \sim\mu\sim y$ imply $x = y$, which is the Moisil determination principle.

If we note $x * y = (x \cap \sim\mu y) \cup (\sim\mu\sim x \cap \sim y)$, we shall prove that $\langle A, 0, *, \sim \rangle$ is a L-algebra.

Lemma 1. $x * y = 0 \rightleftarrows x \leq y$.

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1) We note $0 = \sim 1$.