151. *A Generalization of Curry's Theorem*

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1. Introduction. It is well-known that [3] Glivenko obtained a reduction of the classical proposition logic ***LKS*** to the intuitionistic proposition logic ***LJS*** by putting *double negation* in front of each proposition. Thereafter, [1] Curry, as generalization of the Glivenko theorem above, proved:

$$\vdash_{\boldsymbol{LKS}}\mathfrak{A} \text{ if and only if } \vdash_{\boldsymbol{LJS}}\neg\mathfrak{A}\rightarrow\mathfrak{A};$$
$$\vdash_{\boldsymbol{LDS}}\mathfrak{A} \text{ if and only if } \vdash_{\boldsymbol{LMS}}\neg\mathfrak{A}\rightarrow\mathfrak{A},$$

where ***LM*** is the minimal logic introduced by [5] Johansson which has one axiom $(\mathfrak{A}\rightarrow\mathfrak{B})\rightarrow((\mathfrak{A}\rightarrow\neg\mathfrak{B})\rightarrow\neg\mathfrak{A})$ for negation, and ***LD*** is the logic obtained from ***LM*** by assuming further $\mathfrak{A}\vee\neg\mathfrak{A}$, or $(\neg\mathfrak{A}\rightarrow\mathfrak{A})\rightarrow\mathfrak{A}$ (see [2] Curry).

[6] Kleene[1] and [7] Kuroda generalized the Glivenko theorem to predicate logics, namely to a reduction of the classical predicate logic ***LK*** to the intuitionistic predicate logic ***LJ***, essentially by means of *double negation*.

However, the reductions given by them, may be called reductions of ***LK*** to ***LM***. Namely, we can obtain reductions of ***LK*** to ***LM*** by their transformations. On the other hand, the Glivenko theorem does not hold true between ***LKS*** and ***LMS***. Accordingly, it seems natural to ask whether there is a transformation which reduces ***LK*** to ***LJ***, not to ***LM***, and which reduces ***LD*** to ***LM***, as has been done for proposition logics by Curry.

In the following, the authors define a transformation "$_{[\curlywedge]}$", a modification of Curry's transformation ($\mathfrak{A}$ into $\neg\mathfrak{A}\rightarrow\mathfrak{A}$), by means of which we can solve these problems in the affirmative. The authors would like to express their thanks to Prof. K. Ono for his kind guidance and encouragement.[2]

2. Definition of the transformation. The transformation "$_{[\curlywedge]}$" is defined recursively as follows:

(1) If $\mathfrak{P}$ is an elementary formula, $\mathfrak{P}_{[\curlywedge]}\overset{\frown}{=}(\mathfrak{P}\rightarrow\curlywedge)\rightarrow\mathfrak{P}$.

(2) If $\mathfrak{A}$ and $\mathfrak{B}$ are formulas,

$$(\mathfrak{A}\rightarrow\mathfrak{B})_{[\curlywedge]}\overset{\frown}{=}((\mathfrak{A}_{[\curlywedge]}\rightarrow\mathfrak{B}_{[\curlywedge]})\rightarrow\curlywedge)\rightarrow(\mathfrak{A}_{[\curlywedge]}\rightarrow\mathfrak{B}_{[\curlywedge]}),$$

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1) cf. [4] Gödel. In this paper reductions are given for proposition logic and number theory formulated by Herbrand.

2) Our investigation was originally intended to obtain an interpretation of ***LD*** in ***LO*** under significant suggestion of Prof. K. Ono. See [8] Ono.