207. On Compactness in Ranked Spaces

By Yukio SAKAMOTO,*) Hidetake NAGASHIMA,**) and Kin'ichi YAJIMA***)

(Comm. by Kinjirô KUNUGI, M.J.A., Dec. 12, 1967)

In this paper we will give a definition of compactness in the ranked space [1] and will prove some properties in respect of its compactness. We have used the same terminology as that introduced in the paper "On an Equivalence of Convergences in Ranked spaces" [3].

We say that the ranked space R satisfies the axiom (T_2) of separation, if and only if for any distinct points p and q of R there exist disjoint neighborhoods of p and of q respectively having certain ranks.

We say that the ranked space R satisfies the condition (M), if and only if for all points p of R the following condition is satisfied;

(M) if $V(p) \in \mathfrak{V}_\alpha$, $U(p) \in \mathfrak{V}_\beta$, and $\alpha \leq \beta$ then $V(p) \supseteq U(p)$.

Definition. *A subset A of the ranked space R is sequentially compact if and only if every sequence of A has a subsequence which is R-convergent to a point of A.*

Proposition 1. *Let R be the ranked space satisfying the axiom (T_2) of separation and the condition (M). If a sequence $\{p_\alpha\}$ of R is R-convergent, then $\{\lim_\alpha p_\alpha\}$ consists of only a point.*

Proof. Suppose $p, q \in \{\lim_\alpha p_\alpha\}$ and $p \neq q$. Since $p, q \in \{\lim_\alpha p_\alpha\}$, there exist a fundamental sequence $\{V_\alpha(p)\}$ of neighborhoods of p such that $p_\alpha \in V_\alpha(p)$ and a fundamental sequence $\{U_\alpha(q)\}$ of neighborhoods of q such that $p_\alpha \in U_\alpha(q)$. Hence, for all α

$$p_\alpha \in V_\alpha(p) \cap U_\alpha(q). \tag{1}$$

Since R satisfies the axiom (T_2), there exist a neighborhood $V(p)$ of p and a neighborhood $U(q)$ of q such that $V(p) \in \mathfrak{V}_\gamma$, $U(q) \in \mathfrak{V}_\delta$, and $V(p) \cap U(q) = \phi$.

By the condition (M), there exist $V_{\alpha_0}(p)$ and $U_{\alpha_0}(q)$ which are elements of $\{V_\alpha(p)\}$ and $\{U_\alpha(q)\}$ such that $V(p) \supseteq V_{\alpha_0}(p)$ and $U(q) \supseteq U_{\alpha_0}(q)$. Therefore, by (1) $p_{\alpha_0} \in V_{\alpha_0}(p) \cap U_{\alpha_0}(q) \subseteq V(p) \cap U(q)$, that is, $V(p) \cap U(q) \neq \phi$. This contradiction demonstrates that $\{\lim_\alpha p_\alpha\}$ consists of only a point.

Proposition 2. *Let R be the ranked space satisfying the*

---

*) Japan Women's University.
**) Hokkaido University of Education.
***) Japanese National Railways.