204. On the Dimension of Generators of a Polynomial Algebra over the Mod p Steenrod Algebra

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1. It is well known that if $H^*(X; Z_2)$ is a polynomial algebra (possibly truncated) on a generator x of dimension m and $x^2 \neq 0$, then $m=1, 2, 4$, or 8 [1]. If $H^*(X; Z_p)$ is a truncated polynomial algebra on one generator with dimension $2k$ of height $q>p$, then k is a divisor of $p-1$ [3]—[5]. In the above cases the cohomology algebra has only one generator. In this paper, we are concerned with a truncated polynomial algebra with two generators over the mod p Steenrod algebra where p is an odd prime. On a finitely generated truncated polynomial algebra over the Steenrod algebra, E. Thomas and A. Clark have obtained some results [2][6].

By an algebra A over the mod p Steenrod algebra, we mean a commutative and associative graded Z_p algebra A on which the reduced powers and the Bockstein coboundary act just as if A were the cohomology algebra of a space.

Our results are the followings.

Theorem. 1. *Let A be a truncated polynomial algebra of height $q>p$ with even dimensional generators a and b over the mod p Steenrod algebra. We put dim $a=m$, dim $b=n$, and we assume that $0<m\leqq n$ holds. Then, such an algebra only possible if the dimensions m, n satisfy one of the following conditions.*

(a) $m=2i$, $n=2j$, $i\leqq j<p$, *and*

 (i) *i, j are divisors of $p-1$,*

 (ii) *i is a divisor of $p-1$ and j is a divisor of $i+p-1$,*

 (iii) *j is a divisor of $p-1$ and i is a divisor of $j+p-1$, or*

 (iv) *i is a divisor of $j+p-1$ and j is a divisor of $i+p-1$.*

(b) $m=2i$, $n=2(p+i-1)$, *i is a divisor of* $2(p-1)$.

(c) $m=2i$, $n=2\varepsilon p^f$, *i, ε are divisors of $p-1$, $f\geqq 1$,*

(d) $m=2(\delta p^f+i)$, $n=2\varepsilon p^f$, *ε is a divisor of $p-1$, $0<\delta<\varepsilon$, $0<i<p$, $f\geqq 1$,*

(e) $m=2\varepsilon p^f$, $n=2(\varepsilon p^f+p-1)$, *ε is a divisor of $p-1$, $f\geqq 1$,*

(f) $m\equiv 0$, $n\equiv 0$ (mod $2p$).

Remark. If A is a finitely generated truncated polynomial algebra whose generators have fixed even dimensions m and n, then the same conclusion as above holds for this algebra A. The similar