18. Compactness in Ranked Spaces

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It is the purpose of this note to study certain properties of sequentially compact sets in ranked spaces. Throughout this note, we shall always treat ranked spaces with indicator ω_0 ([2] p. 319), and $i, k, m, n, n_0, n_1, \ldots, n_k, \ldots$ will denote non-negative integers.

In a ranked space, for a point-sequence $\{x_n\}_{n=0,1,2,\ldots}$ and for a point x, if we have $x \in \{\lim_n x_n\}$ ([2] p. 319), then the sequence $\{x_n\}$ is said to *r-converge* to x, or the point x is said to be *an r-limit point* of $\{x_n\}$. The symbol $\mathscr{F}(x)$ will denote the collection of all fundamental sequences of neighbourhoods with respect to a point x ([3] p. 551).

Let A be a subset of a ranked space. If every countable sequence $\{x_n\}_{n=0,1,2,\ldots}$ of points of A contains a subsequence r-converging to a point of A, then A is said to be *r-compact*. The set of all points, each of which is an r-limit point of a countable sequence of points of A, is called the *r-closure* of A and denoted by $\mathrm{cl}(A)$. The set A is said to be *r-closed*, if we have $\mathrm{cl}(A)=A$.

We must take care about the r-convergence in a subset A of a ranked space E. The sequence of points of A, r-converging in the space E to a point x of A, r-converges also to x in the induced ranked space A ([3] p. 550), but the converse is not always true.[1]

Example 1. The interval $I=[-2,2]$ of real numbers with families $\mathfrak{V}_n(x)=\left\{\left(x-\frac{1}{n}, x+\frac{1}{n}\right)\cap I\right\}$ $(x \in I, n=0,1,2,\cdots)$[2] becomes a ranked space with indicator ω_0 which will be denoted by E.
$\left(\text{we put } \frac{1}{0}=+\infty.\right)$

For $x \in I$, let

$$\mathfrak{V}'_n(x)=\begin{cases}\mathfrak{V}_n(x) & \text{when } x\neq 0, \text{ or when } x=0,\ n=0.\\ \left\{\left(-\frac{1}{n}, \frac{1}{n}\right), \left(-2+\frac{1}{n}, 2-\frac{1}{n}\right)\right\} & \text{when } x=0,\ n>0.\end{cases}$$

Then I with $\mathfrak{V}'_n(x)$ $(x \in I, n=0,1,2,\cdots)$ also becomes another ranked space with indicator ω_0 which will be denoted by E'.

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1) A condition which makes the converse hold was given in [6] Proposition 15.

2) $\mathfrak{V}_n(x)$ will denote the family of neighbourhoods of point x and of rank n. See [5] p. 616.