14. On Equivalences of Laws in Elementary Protothetics. II

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In our previous paper [1], we have proved the equivalences of the two laws (i.e., the law of development and the law on the limit of a function).

In this paper, we shall prove the equivalence of the theorems (a) and (a′) which have been called the *generalized law on the limit of a function*. The rules of inference, substitution and replacement used in the systems of elementary protothetics has in detail given in J. Słupecki [2], and our paper [1].

(a) $[f, q]\{[p]\{f(p)\} \equiv f(q)\cdot f(\sim(q))\}$,

(a′) $[f, r, s]\{[p, q]\{f(p, q)\} \equiv f(r, s)\cdot f(r, \sim(s))\cdot f(\sim(r), s)\cdot f(\sim(r), \sim(s))\}$.

To show the equivalence mentioned above, we shall first prove the following theorem.

Theorem 1. $[f, r, s]\{[f, q]\{[p]\{f(p)\} \supset f(q)\cdot f(\sim(q))\}\cdot[u, v]\{f(u, v)\} \supset f(r, s)\cdot f(r, \sim(s))\cdot f(\sim(r), s)\cdot f(\sim(r), \sim(s))\}$.

Proof. (1) $[f, q]\{[p]\{f(p)\} \supset f(q)\cdot f(\sim(q))\}$,

(2) $[u, v]\{f(u, v)\} \supset$

by replacing the variables u, v in the assumption (2) with a variables r, s, we obtain the following expression:

(3) $f(r, s)$. (2)

By a similar procedures, we obtain the following expression:

(4) $f(r, \sim(s))$, (2)

(5) $f(\sim(r), s)$, (2)

(6) $f(\sim(r), \sim(s))$, (2)

$$f(r, s)\cdot f(r, \sim(s))\cdot f(\sim(r), s)\cdot f(\sim(r), \sim(s)).$$

To obtain the consequent we have used the following theorem of the propositional calculus:

$$[p, q, r, s]\{p \supset (q \supset (r \supset (s \supset p\cdot q\cdot r\cdot s)))\},$$

therefore we complete the proof of Theorem 1.

Theorem 2. $[f, q]\{[f, r, s]\{[p, q]\{f(p, q)\} \supset f(r, s)\cdot f(r, \sim(s))\cdot f(\sim(r), s)\cdot f(\sim(r), \sim(s))\}\cdot[u]\{f(u)\} \supset f(q)\cdot f(\sim(q))\}$.

Proof. (1) $[f, r, s]\{[p, q]\{f(p, q)\} \supset f(r, s)\cdot f(r, \sim(s))\cdot f(\sim(r), s)\cdot f(\sim(r), \sim(s))\}$,

(2) $[u]\{f(u)\} \supset$

By replacing the variable u in the assumption (2) with a variable q,