13. *On Axioms of Ontology*

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It is well known that the following expression can act the only axiom of ontology [1], [2]:

(α) $x \varepsilon X \equiv [\exists y]\{y \varepsilon x \wedge y \varepsilon X\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$.

The proof of this based on the following axiom of ontology has been given in " S. Leśniewski's Calculus of Names " by J. Slupecki [2]:

$T1.1.$ $x \varepsilon X \equiv [\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$.

In this paper we shall give *the proof of* $T1.1$ *based on* (α).

The proofs of theorems will be given in the form of suppositional proofs used by J. Slupecki.

(I) $x \varepsilon X \wedge y \varepsilon x \supset x \varepsilon x$.

Proof. (1) $x \varepsilon X$ } {premise}
(2) $y \varepsilon x$ }
(3) $y \varepsilon x \wedge y \varepsilon x$ {2}
(4) $[\exists y]\{y \varepsilon x \wedge y \varepsilon x\}$ {$D\Sigma$: 3}
(5) $[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$ {α, 1}
$x \varepsilon x$ {α, 4, 5}

(II) $x \varepsilon X \wedge y \varepsilon x \supset x \varepsilon y$.

Proof. (1) $x \varepsilon X$ } {premise}
(2) $y \varepsilon x$ }
(3) $[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$ {α, 1}
(4) $x \varepsilon x \wedge y \varepsilon x \supset x \varepsilon y$ {$O\Pi$: 3}
(5) $x \varepsilon x$ {I, 1, 2}
$x \varepsilon y$ {4, 5, 2}

(III) $x \varepsilon X \wedge y \varepsilon x \supset y \varepsilon X$.

Proof. (1) $x \varepsilon X$ } {premise}
(2) $y \varepsilon x$ }
(3) $[x, z]\{x \varepsilon y \wedge z \varepsilon y \supset x \varepsilon z\}$ {α, 2}
(4) $x \varepsilon y$ {II, 1, 2}
(5) $x \varepsilon y \wedge x \varepsilon X$ {4, 1}
(6) $[\exists x]\{x \varepsilon y \wedge x \varepsilon X\}$ {$D\Sigma$: 5}
$y \varepsilon x$ {α, 6, 3}

(IV) $x \varepsilon X \supset [y]\{y \varepsilon x \supset y \varepsilon X\}$.

Proof. (1) $x \varepsilon X$ {premise}
(2) $y \varepsilon x \supset y \varepsilon X$ {III, 1}
$[y]\{y \varepsilon x \supset y \varepsilon X\}$ {$D\Pi$: 2}

(V) $x \varepsilon X \supset [\exists y]\{y \varepsilon x\}$.

Proof. (1) $x \varepsilon X$ {premise}