10. *Simple Type Theory of Gentzen Style with the Inference of Extensionality*

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The definitions of types and expressions are the same as in [2]. We also refer to expressions of type 1 as formulas. As in *LK* or *GLC*, the form

$$A_1, \cdots, A_m \rightarrow B_1, \cdots, B_n,$$

where $A_1, \cdots, A_m, B_1, \cdots, B_n (m, n \geqq 0)$ are formulas, is called a sequent.

The inference rules of our system are as follows:

(I) Structual inference rules

$$\frac{\Gamma \rightarrow \Delta}{A, \Gamma \rightarrow \Delta} \qquad \frac{\Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta, A}$$

$$\frac{A, A, \Gamma \rightarrow \Delta}{A, \Gamma \rightarrow \Delta} \qquad \frac{\Gamma \rightarrow \Delta, A, A}{\Gamma \rightarrow \Delta, A}$$

$$\frac{\Gamma, A, B, \Pi \rightarrow \Delta}{\Gamma, B, A, \Pi \rightarrow \Delta} \qquad \frac{\Gamma \rightarrow \Delta, A, B, \Lambda}{\Gamma \rightarrow \Delta, B, A, \Lambda}$$

(Cut) $$\frac{\Gamma \rightarrow \Delta, A \quad A, \Pi \rightarrow \Lambda}{\Gamma, \Pi \rightarrow \Delta, \Lambda}$$

(II) Inference rules on logical symbols

$$\frac{\Gamma \rightarrow \Delta, A}{\neg A, \Gamma \rightarrow \Delta} \qquad \frac{A, \Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta, \neg A}$$

$$\frac{A, \Gamma \rightarrow \Delta \quad B, \Gamma \rightarrow \Delta}{A \vee B, \Gamma \rightarrow \Delta}$$

$$\frac{\Gamma \rightarrow \Delta, A}{\Gamma \rightarrow \Delta, A \vee B} \qquad \frac{\Gamma \rightarrow \Delta, B}{\Gamma \rightarrow \Delta, A \vee B}$$

$$\frac{A(a^\tau), \Gamma \rightarrow \Delta}{\exists x^\tau A(x^\tau), \Gamma \rightarrow \Delta},$$

where a^τ does not occur in the lower sequent.

$$\frac{\Gamma \rightarrow \Delta, A(e^\tau)}{\Gamma \rightarrow \Delta, \exists x^\tau A(x^\tau)},$$

where e^τ is an arbitrary expression of type τ.

(III) Inference of comprehension

$$\frac{A(e_1^{\tau_1}, \cdots, e_n^{\tau_n}), \Gamma \rightarrow \Delta}{(e_1^{\tau_1}, \cdots, e_n^{\tau_n} \in \lambda x_1^{\tau_1} \cdots x_n^{\tau_n} A(x_1^{\tau_1}, \cdots, x_n^{\tau_n})), \Gamma \rightarrow \Delta}$$

$$\frac{\Gamma \rightarrow \Delta, A(e_1^{\tau_1}, \cdots, e_n^{\tau_n})}{\Gamma \rightarrow \Delta, (e_1^{\tau_1}, \cdots, e_n^{\tau_n} \in \lambda x_1^{\tau_1} \cdots x_n^{\tau_n} A(x_1^{\tau_1}, \cdots, x_n^{\tau_n}))}$$

(IV) Inference of extensionality