35. Note on the Nuclearity of Some Function Spaces. II

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In this note, by the same method as [3], we shall prove the nuclearity of $Z_\Omega\{M_A\}$, which it is introduced by I. M. Gelfand and G. E. Shilov [2].

Definition. Let A be any index set and we assume that, for each element α of A, $M_\alpha(z)$ is a real valued continuous function defined on a open subset Ω of the complex space C^n and it satisfies the following condition: for each $\alpha \in A$, $M_\alpha(z)$ is positive and

$$\text{if } \alpha \leqq \beta \quad \text{then} \quad M_\alpha(z) \leqq M_\beta(z)\ .$$

We put
$$\|\psi\|_\alpha = \sup_{z\in\Omega} M_\alpha(z)\,|\psi(z)| \qquad (\alpha \in A) \tag{1}$$
where ψ is a element of the set of all entire functions on Ω. Then we denote by $Z_\Omega\{M_A\}$ the set of all the entire functions ψ which satisfies $\|\psi\|_\alpha < \infty$ for all $\alpha \in A$ and the topology of $Z_\Omega\{M_A\}$ be defined by the sequence of norms $\|\psi\|_\alpha (\alpha \in A)$.

We shall prove below that $Z_\Omega\{M_A\}$ is a nuclear space if the following two conditions are satisfied.

(N_1^0) For any element α of A there exists an index $\beta \geqq \alpha$ such that $\dfrac{M_\alpha(z)}{M_\beta(z)}$ is integrable on Ω and if Ω is an unbounded open subset then $\lim\limits_{|z|\to\infty} \dfrac{M_\alpha(z)}{M_\beta(z)} = 0$.

(N_2^0) For any index $\alpha \in A$ there exists an index $\beta \geqq \alpha$ such that, for some positive number γ, if $|w-z| \leqq \gamma$ then
$$\frac{M_\alpha(z)}{M_\beta(w)} \leqq C_\alpha \tag{2}$$
where C_α is a constant number depending on α.

Lemma. *If the condition (N_1^0) holds then the initial topology of the space $Z_\Omega\{M_A\}$ is equivalent to the topology introduced by the sequence of semi-norms*
$$\|\psi\|_{\alpha,K} = \sup_{z\in K}\{M_\alpha(z)\,|\psi(z)|\} \qquad \textit{for } \psi \in Z_\Omega\{M_A\}\ , \tag{3}$$
where α be any index in A and K runs all compact subset of Ω.

Proof. Clearly for any $\psi \in Z_\Omega\{M_A\}$
$$\|\psi\|_{\alpha,K} \leqq \|\psi\|_\alpha \tag{4}$$
for all $\alpha \in A$ and compact subset K of Ω.

Next, when Ω is unbounded, for each $\alpha \in A$ and $\psi \in Z_\Omega\{M_A\}$
$$\lim_{|z|\to\infty} M_\alpha(z)\psi(z) = 0\ .$$