33. *Infinite Boundary Value Problem*

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In the usual Dirichlet problem the boundary function is supposed to be finitely continuous or at most to have a small set on which it is infinite. We are interested here in the other extreme where the boundary function is constantly infinite, and report a sufficient condition for the solvability of this boundary value problem, the detail of which will be published elsewhere.

Our problem is formulated as follows. Let M be an m-dimensional orientable C^∞ manifold with a smooth compact boundary α and the ideal boundary β. Here α *may be void* but β is always assumed to be nonempty and isolated from α. The ideal boundary β can be realized topologically in many ways but we do not specify it other than the supposition that $M\cup\alpha\cup\beta$ is a compactification of M.

Consider the elliptic differential operator L given on $M\cup\alpha$ in terms of local coordinate as follows:

$$Lu(x)=\frac{1}{\sqrt{a(x)}}\frac{\partial}{\partial x^i}\left(\sqrt{a(x)}\,a^{ij}(x)\frac{\partial u(x)}{\partial x^j}\right)+b^i(x)\frac{\partial u(x)}{\partial x^i}+c(x)u(x)$$

where $(a^{ij}(x))$ and $(b^i(x))$ $(i,j=1,\cdots,m)$ are contravariant tensors on $M\cup\alpha$, $(a^{ij}(x))$ is strictly positive definite at each $x\in M\cup\alpha$, and $a(x)=\det(a^{ij}(x))^{-1}$.

Here $a^{ij}(x)$, $\partial a^{ij}(x)/\partial x^k$, and $b^i(x)$ are totally differentiable; $\partial^2 a^{ij}(x)/\partial x^k\partial x^l$, $\partial b^i(x)/\partial x^k$, and $c(x)$ are locally uniformly Hölder continuous $(i,j,k,l=1,\cdots,m)$.

We assume that $c(x)\leq 0$ on M and moreover that $c(x)\not\equiv 0$ on M if $\alpha=\phi$. Under these assumptions there exists the Green's function $G(x,y)$ on M for the operator L_x, i.e. the smallest positive fundamental solution for L_x. In terms of the Green's function we can state

Theorem. *Suppose the existence of a subset N of M such that $N\cup\beta$ is a neighborhood of β in $M\cup\alpha\cup\beta$ and*

$$\sup_{(x,y)\in N\times N}\frac{G(x,y)}{G(y,x)}<\infty \tag{1}$$

and

$$\inf_{x\in N} G(x,y)>0 \tag{2}$$

are valid. Then there exists a continuous function u on $M\cup\alpha\cup\beta$