30. On the Representations of SL(3, C). II

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1. Considering the interwinning operators of the representations $\mathfrak{R}(\chi)=\{T^{\chi},\mathfrak{D}_{\chi}\}(\chi=(\lambda_1, \mu_1; \lambda_2, \mu_2))$ of $G=SL(3, \boldsymbol{C})$ discussed in the part I of this work [1], we can obtain the equivalence relations and semi-reduciblity of these representations in the present paper. We shall consider three cases separately: (i) Neither one of pairs $(\lambda_k, \mu_k)(k=1, 2)$ nor $(\lambda_1+\lambda_2, \mu_1+\mu_2)$ is a pair of integers. (ii) Only one of them is a pair of integers. (iii) All of them are pairs of integers.
We shall consider unitary representations in the next paper.

During the preparation of this paper, the author found that Zhelobenko [2] published important results which concern this part of the present works. In this paper we shall consider representations in concrete form.

2. Let $(\lambda_1, \mu_1)=(l_1, m_1)$ be a pair of positive integers, then in the space $\mathfrak{D}_{\chi}$ there exists an invariant subspace $\mathcal{E}_{\chi}^1$ which is the linear subspace of polynomials in z_1 and $z_1z_2-z_3$ of degree (l_1-1, m_1-1) with coefficients $a_{pq}(z_2, z_3)$ which are such C^{∞}-functions that $z_2^{(\lambda_2-1,\mu_2-1)}a_{pq}(1/z_2, z_3/z_2)$ and $z_3^{(\lambda_2-1,\mu_2-1)}a_{pq}(z_2/(z_3, 1/z_3)$ are again C^{∞}-functions.

Then we have

Theorem 1. (1°) *If neither one of pairs* (λ_k, μ_k) nor $(\lambda_1+\lambda_2, \mu_1+\mu_2)$ *is a pair of integers of the same signature, then the representation* $\mathfrak{R}(\chi)$ *is completely irreducible.*

(2°) *If* $(\lambda_1, \mu_1)=(l_1, m_1)$ *is a pair of positive integers and* (λ_2, μ_2) *is not a pair of integers of the same signature, then the representation* $\{T^{\chi}, \mathcal{E}_{\chi}^1\}$ *is completely irreducible.*

We omit the proof of this theorem. The principal purpose of the present paper is to prove the following theorem in section 3, 4, and 5:

Theorem 2. *The representations* $\mathfrak{R}(\chi)$, $\mathfrak{R}(\chi')$ *of* G *are equivalent or partially equivalent if and only if* $\chi'=\chi^s$ *for some* $s\in W$.

3. *Case* (*i*) (*non-degenerate case*). There are six intertwinning operators from $\mathfrak{R}(\chi)$ to $\mathfrak{R}(\chi^s)$:

$$A_1\varphi(z)=\gamma(\lambda_1, \mu_1)\int z_1'^{(-\lambda_1-1,-\mu_1-1)}\varphi(z_1'+z_1, z_2, z_3)dz_1',$$

$$A_2\varphi(z)=\gamma(\lambda_2, \mu_2)\int z_2'^{(-\lambda_2-1,-\mu_2-1)}\varphi(z_1, z_2'+z_2, z_1z_2-z_3)dz_2',$$