77. *On Submanifolds in Spaces of Constant and Constant Holomorphic Curvatures*

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1. Fundamental formulas. Let M and $\overline{M}$ be two Riemannian manifolds of dimension n and $n+m$ respectively, with M immersed in $\overline{M}$. We shall denote $\langle\ ,\ \rangle$ the Riemannian metric of $\overline{M}$ and $\overline{\nabla}$ the Riemannian connection of $\overline{M}$ associated with this metric. Let us also denote $\langle\ ,\ \rangle$ the induced Riemannian metric of M. Let $V(M)$ be the ring of the differentiable vector fields on M, $NV(M)$ be the collection of normal vector fields to M defined on a proper open subset of M, which is spanned by mutually orthogonal m unit normal vector fields $C_1, \cdots, C_m$.

Let
$$p: \quad V(M)+NV(M)\rightarrow V(M)$$
be a natural projection.

For X in $V(M)$, we put

$$p\overline{\nabla}_X C_i = -A_i X. \qquad (i=1, \cdots, m) \tag{1.1}$$

Proposition 1.1. For X, Y in $V(M)$, we have

$$\overline{\nabla}_X Y = \nabla_X Y + \sum_{i=1}^{m} \langle A_i X, Y\rangle C_i \quad \text{where} \quad \nabla_X Y \quad \text{in} \quad V(M). \tag{1.2}$$

(1.3) ∇ is a Riemannian connection of M associated with the induced Riemannian metric and A_i are self-adjoint (1, 1) type tensors.

Proof. We may set

$$\overline{\nabla}_X Y = \nabla_X Y + \sum_{i=1}^{m} f_i C_i. \tag{1.4}$$

Then, since $\langle Y, C_i\rangle = 0$, differentiating covariantly, we get

$$\langle \overline{\nabla}_X Y, C_i\rangle + \langle Y, \overline{\nabla}_X C_i\rangle = 0. \tag{1.5}$$

Substituting (1.4) into (1.5) leads to

$$f_i = \langle A_i X, Y\rangle. \tag{1.6}$$

The properties of (1.3) can be easily checked. Q.E.D.

Let $\{E_1, \cdots, E_n\}$ be an orthonormal basis on an open subset of M. We put

$$H = \sum_{i=1}^{m} (\text{tr } A_i) C_i \tag{1.7}$$

where tr denotes the trace, $\text{tr } A_i = \sum_{\alpha=1}^{n} \langle A_i E_\alpha, E_\alpha\rangle$. H is called the mean curvature vector field of M. A submanifold M is called minimal if $\text{tr } A_i = 0$, totally geodesic if $A_i = 0$ and totally umbilical if $\langle A_i X, X\rangle$