68. *A Minimal Property for an Operator of Hilbert-Schmidt Class*

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1. If T is a completely continuous operator defined on a Hilbert space H, then T can be expressed in the *Schatten formula*:

$$T=\sum_{i=1}^{\infty}\lambda_i\varphi_i\otimes\psi_i, \tag{1}$$

where (i) $\{\lambda_i\}$ is a decreasing sequence of positive numbers which are proper values of

$$|T|=(T^*T)^{\frac{1}{2}}, \tag{2}$$

(ii) $\{\varphi_i\}$ and $\{\psi_i\}$ are orthonormal sets in H, and (iii) a *dyad* $f\otimes g$ is defined by

$$(f\otimes g)h=(h\,|\,g)f, \tag{3}$$

for every $h\in H$, cf. [2]. Since the proper values of a completely continuous operator $|T|$ converge to zero, the series of (1) converges uniformly.

An operator T acting on H is of *Hilbert-Schmidt class* if

$$||T||_2^2=\sum_{i=1}^{\infty}||T\phi_i||^2 \tag{4}$$

is finite whenever $\{\phi_i\}$ is a orthonormal base of H. An operator T of Hilbert-Schmidt class is completely continuous and

$$||T||_2^2=\sum_{i=1}^{\infty}\lambda_i^2, \tag{5}$$

where $\{\lambda_i\}$ is the coefficients of the Schatten formula (1).

The purpose of the present note is to show the following minimal property of the Schatten formula:

Theorem 1. *If T is of Hilbert-Schmidt class and expressed in* (1), *then*

$$||T-\lambda_1\varphi_1\otimes\psi_1||_2$$

attains its minimum among all approximation by dyads: that is,

$$||T-\lambda_1\varphi_1\otimes\psi_1||_2\leqq||T-f\otimes g||_2, \tag{6}$$

for every dyad $f\otimes g$.

2. Let $H=L^2[0,1]$. If $u(x,y)$ is a measurable function defined on $[0,1]\times[0,1]$ with

$$||u||^2=\int_0^1\int_0^1|u(x,y)|^2dx\,dy<+\infty,$$

then, for every $f\in H$,