108. On the Hölder Continuity of Stationary Gaussian Processes

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Let $X=\{X(t); -\infty<t<\infty\}$ be a real, separable and stochastically continuous stationary Gaussian process with mean zero and with the covariance function $\rho(t)=E(X(t+s)X(s))$. Without loss of generality, we may assume $\rho(0)=1$. The continuity of path functions of X has been studied by many authors and further, under the rather strong condition on $\sigma^2(t)=E((X(t+s)-X(s))^2)=2(1-\rho(t))$, the Hölder continuity of $X(t, w)$[1] was discussed by Yu. K. Belayev in his [1], among others. Our purpose in this paper is to give the final result about the Hölder continuity of $X(t, w)$ under the similar conditions to Belayev's one. In the case of Brownian motion with d-dimensional parameter, the same problem was solved by T. Sirao [3]. We will state our result in the form corresponding to the Brownian case. After the Brownian case, we first introduce the notions of the upper class and lower class for $\{X(t); 0\leqq t\leqq 1\}$. If there exists a positive number δ such that $|t-s|\leqq\delta$ $(0\leqq t, s\leqq 1)$ implies

$$|f(t)-f(s)|\leqq g(|t-s|),$$

then we say that $f(t)$ satisfies Lipschitz's condition relative to $g(t)$. Let $\varphi(t)$ be a positive, non-decreasing and continuous function defined for large t's. If almost all sample functions $X(t, w)$ satisfy (do not satisfy) Lipschitz's condition relative to $g(t)=\sigma(t)\varphi(1/t)$, then we say that $\varphi(t)$ belongs to the upper (lower) class with respect to the uniform continuity of $\{X(t); 0\leqq t\leqq 1\}$ and denote it by $\varphi\in\mathcal{U}^u(\mathcal{L}^u)$.

Next, we consider following Condition (A) consisting in (A. 1) and (A. 2).

(A. 1) There exist constants $0<\alpha<2$, $-\infty<\beta<\infty$, and $\delta>0$ such that for any h in $(0, \delta)$

$$C_1\frac{h^\alpha}{|\log h|^\beta}\leqq\sigma^2(h)\leqq C_2\frac{h^\alpha}{|\log h|^\beta},\quad 0<C_1<C_2<\infty.$$

(A. 2) $\sigma^2(h)$ is concave in $(0, \delta)$ if either one of $0<\alpha<1$, $-\infty<\beta<\infty$ or $\alpha=1$, $\beta\leqq 0$ holds and $\sigma^2(h)$ is convex in $(0, \delta)$ if either one of $\alpha>1$, $-\infty<\beta<\infty$ or $\alpha=1$, $\beta\geqq 0$ holds, where α, β, γ are constants mentioned in (A. 1).

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1) w denotes a probability parameter.