107. σ-Spaces and Closed Mappings. II

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1. This is the continuation of our previous paper [6] in which we proved the following:

Theorem. *Let X be a normal T_1 σ-space and f a closed mapping*[1] *of X onto a topological space Y. Then Y is a normal T_1 σ-space such that the set $\{y \mid \partial f^{-1}(y)$ is not countably compact$\}$ is σ-discrete in Y, where $\partial f^{-1}(y)$ denotes the boundary of $f^{-1}(y)$.*

The purpose of this paper is to consider some applications of the above theorem to σ_0 spaces and to prove three theorems below. We shall say that a topological space X is *countable-dimensional* or σ_0 if it is the sum of X_i, $i=1, 2, \cdots$, with $\dim X_i \leqq 0$, where $\dim X_i$ denotes the covering dimension of X_i defined by means of finite open coverings, and that X is *uncountable-dimensional* if it is not σ_0.

Theorem 1. *Let X be a collectionwise normal T_1 σ-space and f a closed mapping of X onto a topological space Y such that $\partial f^{-1}(y)$ is countable for each $y \in Y$ or discrete for each $y \in Y$. Then Y is a countable sum of subspaces, each of which is homeomorphic to a subspace of X.*

Theorem 2. *Let X be a collectionwise normal σ_0 and T_1 σ-space and f a closed mapping of X onto an uncountable-dimensional space Y. Then Y contains an uncountable-dimensional subset N of Y such that $\partial f^{-1}(y)$ is uncountable for each $y \in Y$.*

Theorem 3. *Let X be a collectionwise normal σ_0 and T_1 σ-space and f a closed mapping of X onto an uncountable-dimensional space Y. Then Y contains an uncountable-dimensional subset Y such that $\partial f^{-1}(y)$ is dense-in-itself, non-empty and compact for each $y \in Y$.*

The first two theorems are generalizations of the results obtained by A. Arhangel'skii [2] which were proved in the case of spaces with countable nets and the last one is a generalization of K. Nagami's theorem [4] which was proved in the case of metric space, all of which concerned with a problem of P. Alexandroff [1] on the effect of closed mappings on countable-dimensional spaces.

2. To prove our results we need a few preliminaries.

Lemma 1. *Let $\mathfrak{F}$ be a collection of subsets of a topological space*

1) All mappings in this paper are *continuous*.