105. On Generalized Commuting Properties of Metric Automorphisms. I

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Adler [1] has proved that the generalized commuting order of a totally ergodic automorphism on a compact metric abelian group is two. We shall prove that the generalized commuting order of a totally ergodic metric automorphism on the measure algebra associated with a finite measure space is two. The study in this paper depends on Adler's idea in [1].

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Let (X, Σ, m) be a finite measure space where X is a set of elements, Σ a σ-field of measurable subsets of X, and m a finite measure on Σ. A measure algebra associated with the measure space (X, Σ, m) is the Boolean algebra formed by identifying sets in Σ whose symmetric difference has measure zero. An automorphism of the measure algebra is called a *metric automorphism*. Let G be the group of all metric automorphisms on the measure algebra with the identity I. $C_n(T)$, $n=1, 2, \cdots$ of subfamilies of G associated with a metric automorphism T are defined inductively as follows:

$$C_0(T)=\{I\},$$
$$C_n(T)=\{S \in G : STS^{-1}T^{-1} \in C_{n-1}(T)\}, \quad n=1, 2, \cdots.$$

If there exists an integer N such that $C_N(T)=C_{N+1}(T)$ then $C_n(T)=C_{n+1}(T)$ for all $n \geqslant N$ and in this case we define $N(T)=\min\{N : C_N(T)=C_{N+1}(T)\}$ and otherwise $N(T)=\infty$. $N(T)$ is called the *generalized commuting order* of T. Let $L^2(X)$ be the Hilbert space of complex-valued square integrable functions defined on (X, Σ, m) and $L^\infty(X)$ the Banach space of complex-valued m essentially bounded functions defined on (X, Σ, m). A metric automorphism T is said to have *discrete spectrum* if there is a basis $\mathbf{0}$ of $L^2(X)$ each term of which is a normalized proper function of the linear isometry V_T induced by T. Clearly $\mathbf{0}$ includes the circle group K in the complex plane. If T is ergodic then it turns out that $|f|=1$ a.e. for each $f \in \mathbf{0}$ and that $\mathbf{0}=\mathbf{0}(T)\times K$ where $\mathbf{0}(T)$ is a subgroup of $\mathbf{0}$ isomorphic to the factor group $\mathbf{0}/K$ [4]. If f is a proper function of T and α its proper value, then we denote by $\alpha_T(f)$ the proper value α. T is said to be