104. A Remark on the Normal Expectations

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1. As concerns the channels in the mathematical theory of information, we have discussed under operator method and have introduced the notion of generalized channel in [3].

In this paper, we shall show a relation between certain generalized channels and normal expectations; that is, the conjugate mapping of a generalized channel having a special property is a normal expectation and the converse is true. Furthermore, by using this result, we shall study that for which type von Neumann algebra $\mathcal{A}$ on a Hilbert space $\mathfrak{H}$ there exists a faithful normal expectation of full operator algebra $L(\mathfrak{H})$ onto $\mathcal{A}$.

2. Consider a von Neumann algebra $\mathcal{A}$, denote the conjugate space as $\mathcal{A}^*$ and the subconjugate space of all ultraweakly continuous linear functionals on $\mathcal{A}$ as $\mathcal{A}_*$ basing on the definition of Dixmier [2].

Let $\mathcal{A}$ and $\mathcal{B}$ be two von Neumann algebras, then a positive linear mapping π of $\mathcal{A}_*$ into $\mathcal{B}_*$ is called a *generalized channel* if π preserves the norm of positive elements. Then the following proposition is obtained in [3].

Proposition 1. *A positive linear mapping π of $\mathcal{A}_*$ into $\mathcal{B}_*$ is a generalized channel if and only if the conjugate mapping π^* is a positive normal linear mapping of $\mathcal{B}$ into $\mathcal{A}$ preserving the identity.*

Let $\mathcal{A}$ be a von Neumann algebra and $\mathcal{B}$ a von Neumann subalgebra of $\mathcal{A}$, then the positive linear mapping e of $\mathcal{A}$ onto $\mathcal{B}$ is called an expectation of $\mathcal{A}$ onto $\mathcal{B}$ if e satisfies the following equalities:

(1) $$I^e = I$$

(2) $$(BA)^e = BA^e \quad \text{for all } A \in \mathcal{A} \text{ and } B \in \mathcal{B}.$$

Define the operator L_A on $\mathcal{A}^*$ for each $A \in \mathcal{A}$ such that

(3) $$L_A f(X) = f(AX) \quad \text{for all } f \in \mathcal{A}^* \text{ and } X \in \mathcal{A},$$

then we have following theorem.

Theorem 2. *Let $\mathcal{A}$ be a von Neumann algebra and $\mathcal{B}$ a von Neumann subalgebra of $\mathcal{A}$, then a mapping π of $\mathcal{B}_*$ to $\mathcal{A}_*$ is a generalized channel and*

(4) $$\pi L_B = L_B \pi \quad \textit{for any } B \in \mathcal{B}$$

if and only if the conjugate mapping π^* of $\mathcal{A}$ to $\mathcal{B}$ is a normal expectation.