97. On Screenable Topological Spaces

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In recent years a number of papers, notably [2], [7], and [13], have been at least partially concerned with screenability in topological spaces and the interrelations between various generalized compactness properties and screenability. In this note it is shown that both screenability and strong screenability are intermediate to, and different from, certain generalized Lindelöf properties introduced in [6]. Also it is proved that in screenable spaces, countable metacompactness, countable paracompactness and countable compactness are equivalent to metacompactness, paracompactness and compactness, respectively. The latter result generalizes theorems of Heath [7] and the author [6].

Throughout this paper, *no* separation axiom (e.g., the T_1-axiom) is assumed tacitly for the topological spaces under discussion. All terminology is consistent with that used in [4] and [6]. The properties of screenability and strong screenability were first defined by Bing [2].

Definition 1. A collection $\mathcal{C}$ of subsets of a topological space X is

(i) *σ-pairwise-disjoint* if and only if $\mathcal{C}$ is the union of countably many collections each of which is a pairwise-disjoint collection of subsets of X.

(ii) *discrete* if and only if $\{\bar{C}: C \in \mathcal{C}\}$ is pairwise-disjoint and the union of any subcollection of $\{\bar{C}: C \in \mathcal{C}\}$ is closed in X.

(iii) *σ-discrete* if and only if $\mathcal{C}$ is the union of countably many discrete collections of subsets of X.

Definition 2. A topological space X is

(i) *screenable* if and only if each open cover of X has a σ-pairwise-disjoint, open refinement.

(ii) *strongly screenable* if and only if each open cover of X has a σ-discrete, open refinement.

Lemma. *If $\mathcal{R}$ is a star-countable open cover of a topological*

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