184. *Non-existence of Holomorphic Solutions of $\partial u/\partial z_1 = f$*

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(Comm. by Kunihiko KODAIRA, M. J. A., Oct. 12, 1968)

1. Consider the partial differential equation

$$\frac{\partial u}{\partial z_1} = f \tag{1}$$

on a domain D in the complex affine space $C^n(z_1, z_2, \cdots, z_n)$, where the given function $f = f(z_1, z_2, \cdots, z_n)$ is holomorphic in D. We are interested in global holomorphic solutions u of (1).

In particular, for $n=1$, it is well known that (1) has a global holomorphic solution for every f if and only if D is simply connected. We ask whether this is true for $n \geqslant 2$.

In what follows, we shall answer negatively this question. Namely, we shall give a domain D in C^3 which is holomorphically equivalent to a polycylinder (i.e., a product domain of disks) and on which (1) has no global solution for some holomorphic functions f.

For $n \geqslant 2$, a counterpart of simply connected domains is sometimes regarded as Runge domains.*) We shall give, however, a Runge domain $D \subset C^2$ on which (1) has no global solution for some holomorphic functions f.

On a convex domain in C^n, the existence theorem for global solutions of linear partial differential equations with constant coefficients was established by Harvey [2], and it was extended by Komatsu [3] to systems of those satisfying a compatibility condition. However convexity is a stronger condition than simply-connectedness. Moreover, as the case $n=1$ indicates, whether the simply-connectedness is sufficient or not for the existence of global solutions of such differential equations has been unknown for $n>1$.

2. Now we prove a proposition in order to show following Theorem 1.

Proposition. *Let D be a domain of holomorphy in $C^n(z_1, z_2, \cdots, z_n)$. If there exists a complex line L of the form $L = \{(z_1, z_2, \cdots, z_n) \in C^n \mid z_2 = z_2^0, \cdots, z_n = z_n^0\}$ such that the intersection of L and D contains a multiply connected domain (in L), then (1) has no global solution on D for some holomorphic functions f.*

*) A domain of holomorphy in C^n is called a Runge domain if every holomorphic function in the domain can be uniformly approximated on an arbitrary compact set in the domain by polynomials.