183. *Approximation of Semigroups of Operators on Fréchet Spaces*

By A. B. BUCHE
Department of Mathematics, Wayne State University, Detroit, Michigan 48202, U. S. A.

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1. Introduction. Let $\mathfrak{X}$ be a Fréchet space (cf. Treves [12], Chap. 10, pp. 85-94), and let $\mathcal{L}(\mathfrak{X})$ be the space of continuous linear transformations of $\mathfrak{X}$ into itself. Let $\{T(t), t \in R^+\}$, $T(t): R^+ \to \mathcal{L}(\mathfrak{X})$, be a one-parameter family of continuous operators. The family $\{T(t), t \in R^+\}$ is called a *semigroup of operators* if

(1) $$T(s+t)=T(s)\,T(t),\ s,\ t \in R^+,\ T(0)=I.$$

The *infinitesimal generator* of the semigroup $T(t)$ is defined as

(2) $$A = s - \lim_{h\to 0}(T(h)-I)/h,$$

and $\mathcal{D}(A)$ is the set of all $f \in \mathfrak{X}$ for which the above limit exists. The *resolvent operator* is difined as the abstract Laplace transform of $T(t)$, that is

(3) $$R(\lambda\,; A)f=\int_0^\infty e^{-\lambda t}T(t)f\,dt,\ f \in \mathfrak{X}.$$

The theory of semigroups on Fréchet spaces, which is a generalization of the theory of semigroups on Banach spaces, has been developed by Komatsu [5], Mate [6], Miyadera [7], Schwartz [10], and Yosida [14]. The study of the approximation of semigroups on Banach spaces was initiated by Trotter [13] (cf. also Kato [4]). We refer to Hasegawa [2], Ôharu [9] for other results on the aproximation of semigroups on Banach spaces, and to Ôharu [8] and Yosida [14] for the generalization of Trotter's results to locally convex topological vector spaces.

In this paper we state some results on the approximation of semigroups on Fréchet spaces, and consider as a concrete example, the approximation of a semigroup on the Fréchet space of infinitely differentiable functions, utilizing Chlodovsky's [1] generalizations of Bernstein polynomials on an infinite interval. The proofs subsidiary results will be given elsewhere. We remark that Seidman [11] independently obtained some of our results, following the methodology of Yosida.

2. Convergence of semigroups on Fréchet spaces. *Approximation theorems.* In this section we consider a sequence of Fréchet spaces $\{\mathfrak{X}_n\}$, $\mathfrak{X}_1 \subset \mathfrak{X}_2 \subset \cdots \subset \mathfrak{X}_n \subset \mathfrak{X}_{n+1} \subset \cdots$, and a countable family of