225. On the Essential Set of Function Algebras

By Hiroshi ISHIKAWA,*) Jun TOMIYAMA,**)
and Junzo WADA***)

(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 12, 1968)

Let A be a function algebra on a compact Hausdorff space X, that is, A is a closed subalgebra of $C(X)$ which separates the points of X and contains the constants. In the following we shall present several results relating to the essential set of A, some of which are regarded as generalizations of the results published in several literatures [4], [7], and [8]. Complete proofs of these theorems and other details will be published elsewhere.

Throughout this paper M will indicate the maximal ideal space of A. The Šilov boundary of A will be denoted by ∂A. For a subset F in X, we shall denote by $A|F$ the restricted algebra of A to F. If $A|F$ is closed in $C(F)$, $A|F$ is regarded as a function algebra on F. A closed subset F in X is called an interpolation set for A if $A|F=C(F)$, and is called a closed restriction set if $A|F$ is closed in $C(F)$. Let G be an open set in X. G is called a w-interpolation set for A if any compact subset in G is an interpolation set for A.

Theorem 1. *Let A be a function algebra on X and let $A\neq C(X)$. If G is any w-interpolation set for A, then $G\cap\partial_{A|E}=\phi$, where E is the essential set of A in X.*

Corollary. *Let A be a function algebra on X and suppose $E=\partial_{A|E}$. Then the set $X\sim E$ is the largest w-interpolation set for A.*

The hypothesis of the corollary is necessary. Let X be the set consisting of the unit circle and the origin 0 in the unit disc and let A be the restriction of A_0 to X, where A_0 denotes the function algebra of all continuous functions on the closed unit disc which are analytic on the open unit disc. Then $E=X$. But we here see that $G=\{0\}$ is a w-interpolation set and $G\subsetneqq X\sim E=\phi$.

Bishop [3] and Glicksberg [6] have proved that A is characterized by the disjoint closed partitions of its maximal antisymmetric sets and Tomiyama [11] has shown that among these sets the set P of all maximal antisymmetric sets in X consisting of one point is free from the representing space X and plays a special rôle in determining the struc-

*) Ryukyu University, Naha, Okinawa, and Mathematical Institute, Tôhoku University, Sendai.
**) Department of Mathematics, Yamagata University, Yamagata.
***) Department of Mathematics, Waseda University, Tokyo.