38. On Weak Convergence of Transformations in Topological Measure Spaces. II[1)]

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The author extended slightly in [4] a theorem of F. Papangelou [2, Theorem 2] as follows: Let X be a metrizable locally compact space and μ a σ-finite Radon measure on X. Then a sequence $\{T_n\}$ of invertible μ-measure-preserving transformations in X converges to an invertible μ-measure-preserving transformation T in X weakly if and only if every subsequence $\{T_{k(n)}\}$ of $\{T_n\}$ has a subsequence $\{T_{k(l(n))}\}$ which converges to T almost everywhere.

In this paper we study weak convergence of a sequence $\{T_n\}$ of invertible μ-measure-preserving transformations in a metrizable space X with a tight measure μ. The theorems below generalize the first two theorems in [4].

Let (M, Ω, μ) be any measure space. The members of Ω are called measurable. If E is measurable then we will say that the measure space (E, Ω_E, μ_E), where $\Omega_E \equiv \{F \in \Omega \mid F \subset E\}$ and $\mu_E(F) \equiv \mu(F)$ for F which belongs to Ω_E, is a subspace of (M, Ω, μ).

Definition 1. A measure space (M, Ω, μ) is a σ-finite Lebesgue space if there exists a countable family $\Gamma = \{M_n\}$ of mutually disjoint measurable sets such that $0 < \mu(M_n) < \infty$, $\bigcup_n M_n = M$, and $(M_n, \Omega_{M_n}, \mu_{M_n})$ is a Lebesgue space in the sense of V. A. Rohlin [3] for each n.

Proposition 1. *If (M, Ω, μ) is a σ-finite Lebesgue space then there exist a locally compact, σ-compact, metrizable space H containing M and a Radon measure ν on H which satisfy the following properties:*

(i) *M is a ν-measurable subset of H and $\nu(H-M)=0$.*

(ii) *(M, Ω, μ) is a subspace of $(H, \mathfrak{M}, \nu)$, where $\mathfrak{M}$ is the σ-field of subsets of H which are ν-meaurable.*

Moreover if $\Gamma_n = \{\Gamma_{nj}\}$ is a basis of the Lebesgue space $(M_n, \Omega_{M_n}, \mu_{M_n})$, where $\{M_n\}$ is a countable measurable decomposition of M such that $(M_n, \Omega_{M_n}, \mu_{M_n})$ is a Lebesgue space for each n, then the sets of the form $\bigcap_{j=1}^{N} A_{nj}$, where A_{nj} stands for one of the two sets Γ_{nj} and $M_n - \Gamma_{nj}$, can be taken as the topological open basis of the topological subspace

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