33. On Certain Mixed Problem for Hyperbolic Equations of Higher Order

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1. Introduction. Let Ω be the half-space of $\boldsymbol{R}^n: \{(x_1, x_2, \cdots, x_n) \mid x_n>0\}$, and Γ be a boundary of Ω.

Consider the hyperbolic equation

$$Lu=\left(\frac{\partial^{2m}}{\partial t^{2m}}+a_1(x, D)\frac{\partial^{2m-1}}{\partial t^{2m-1}}+\cdots+a_{2m}(x, D)\right)u+B\left(x, D, \frac{\partial}{\partial t}\right)u=f \tag{1.1}$$

where $a_k(x, D)=\sum_{|\alpha|=k} a_\alpha(x)D^\alpha$, $D_j=\frac{1}{\sqrt{-1}}\frac{\partial}{\partial x_j}$, $\alpha=(\alpha_1, \cdots, \alpha_n)$, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $D^\alpha=D_1^{\alpha_1}\cdots D_n^{\alpha_n}$, and B is an arbitrary differential operator of order $(2m-1)$.

We assume that all coefficients are sufficiently differentiable and bounded with their derivatives in $\boldsymbol{R}^n$.

Our aim of the present note is to assert the following

Theorem 1. *We assume that $a_{\alpha_1\cdots\alpha_n}(x', 0)=0$ when α_n is odd. Let all the roots $\tau_i(x, \xi)$, $(i=1, \cdots, 2m)$ with respect to τ of the equation $\tau^{2m}+a_1(x, \xi)\tau^{2m-1}+\cdots+a_{2m}(x, \xi)=0$ be pure imaginary, distinct and not zero,* uniformly. *Then for any $f(t, x)\in C^1([0, T]; L^2(\Omega))$ and any initial data $\left(u(0, x), \frac{\partial u}{\partial t}(0, x), \cdots, \frac{\partial^{2m-1}u}{\partial t^{2m-1}}(0, x)\right)\in \mathcal{D}_i$ $(i=1, 2)$, there exists a unique solution u of the equation* (1.1) *satisfying boundary conditions*

$$u|_\Gamma=\Delta u|_\Gamma=\cdots=\Delta^{m-1}u|_\Gamma=0, \tag{1.2}$$

or

$$\frac{\partial}{\partial x_n}u|_\Gamma=\frac{\partial}{\partial x_n}\Delta u|_\Gamma=\cdots=\frac{\partial}{\partial x_n}\Delta^{m-1}u|_\Gamma=0. \tag{1.3}$$

The solution satisfies $\left(u(t, x), \frac{\partial u}{\partial t}(t, x), \cdots, \frac{\partial^{2m}u}{\partial t^{2m}}(t, x)\right)\in C^0([0, T]; \mathcal{D}_i\times L^2(\Omega))$, where $\mathcal{D}_1=D(\Lambda_-^{2m})\times\cdots\times D(\Lambda_-)$, $\mathcal{D}_2=D(\Lambda_+^{2m})\times\cdots\times D(\Lambda_+)$. In the case of Dirichlet type boundary condition (1.2), we consider $\mathcal{D}_1$, and in the case of Neumann type boundary condition (1.3), we consider $\mathcal{D}_2$. The definitions of Λ_+, Λ_- are represented in the following section.

It is not difficult to show that from the considerations in the proof of Theorem 1 it implies the theorems obtained by S. Mizohata [5]