86. *Periodic Solutions of the Third Sorte for the Restricted Problem of Three Bodies*

By Yoshihide KOZAI
Tokyo Astronomical Observatory

(Comm. by Yusuke HAGIHARA, M. J. A., May 12, 1969)

Abstract. Five periodic solutions with moderate eccentricities and high inclinations for the three-dimensional restricted problem of three bodies are found for cases of 3 : 2, 2 : 1, and 4 : 1 of the mean motions by expanding the disturbing functions by use of a high-speed computer. The expansion with respect to the inclination is made by Tisserand's polynomials and that to the eccentricity is made by Newcomb operators up to the tenth power. The periodic solutions found here correspond to orbits, for which secular and long-periodic perturbations of orbital elements except for the mean anomaly vanish. The existence of such periodic orbits are verified by numerical integration method for a case that the disturbing mass is 0.001.

1. Introduction. Equations of motion for three-dimensional restricted problem of three bodies, that is, for an asteroid moving under gravitational attractions of the Sun and Jupiter on a circular orbit, are written in canonical form by use of the following Delaunay variables:

$$\left.\begin{array}{ll} L=k\sqrt{a}, & l\text{: the mean anomaly,} \\ G=L\sqrt{1-e^2}, & g\text{: the argument of perihelion,} \\ H=G\cos i, & h\text{: the longitude of the ascending node,} \\ & \quad \Omega\text{, minus the longitude of Jupiter,} \end{array}\right\} \qquad (1)$$

where a, e, and i are, respectively, the semi-major axis, the eccentricity, and the orbital inclination to Jupiter's orbital plane for the asteroid, and k is the gravitational constant of Gauss. The units are so chosen that the mass of the Sun and the mean motion and the semi-major axis of Jupiter are unity.

Short-periodic terms which depend on l and/or h can be eliminated from the Hamiltonian by von Zeipel's transformation, for example, and, therefore, the equations of motion are reduced to those of one degree of freedom since L and H are constant after the transformation. Then the values of G can be derived as a function of g by solving the energy integral, and if the inclination is sufficiently high, stationary solutions, in which G and g are constant, are found for $2g=180°$.[1]

However, if the mean motion, n, of the asteroined is nearly com-