79. *Generalizations of M-spaces. II*

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In the previous paper [4] we obtained a characterization of M'-spaces as a generalization of M-spaces and Morita's paracompactification of M'-spaces. In this paper we shall give necessary and sufficient conditions for an M'-space to be M-space and show that the product space of M'-spaces need not be an M'-space and that the property of being M'-space is not necessarily invariant under a perfect mapping (see [2] or [4] for terminologies and notations).

1. Relation between M'- and M-spaces.

A space X is a *cb-space* (resp. *weak cb-space*) if given a decreasing sequence $\{F_n\}$ of closed sets (resp. regular-closed sets) of X with empty intersection, there exists a sequence $\{Z_n\}$ of zero sets with empty intersection such that $F_n \subset Z_n$ for each n where a subset F is *regular-closed* if cl (int F)$=F$.

Lemma 1.1. *The following results has been obtained in* ([5], [6]).

1) *X is a cb-space if and only if X is both countably paracompact and week cb.*

2) *For a pseudocompact space X the followings are equivalent:* i) *X is a cb-space,* ii) *X is countably compact and* iii) *X is countably paracompact.*

3) *A countably compact space is a cb-space.*

4) *A pseudocompact space is a weak cb-space.*

The following lemma is obvious.

Lemma 1.2. *If $\{U_n\}$ is a decreasing sequence of open sets of X such that $\cap \bar{U}_n=\varnothing$, then*

1) *there exists a locally finite discrete collection $\{V_n\}$ of open sets of X such that $\bar{V}_n \subset U_n$ and $\bar{V}_n \cap \bar{V}_m=\varnothing$ $(n\neq m)$,*

2) *there exists a non-negative continuous function f on X such that $f=0$ on $X-\cup V_n$, $0\leq f\leq n$ on V_n and $f(x_n)=n$ for some point x_n of V_n, and*

3) *$\{Z_n ; Z_n=\{x ; f(x)\geq n\}\}$ is a decreasing sequence of zero sets of X with empty intersection.*

Theorem 1.3. *An M'-space is a weak cb-space.*

Proof. Let φ be an SZ-mapping from an M'-space X onto a metric space Y and $\{\mathfrak{V}_i ; i\in N\}$ be a normal sequence of open covering of Y such that $\{\mathrm{St}(y, \mathfrak{V}_i) ; i\in N\}$ is a basis of neighborhoods at each point y