78. Generalizations of M-spaces. I

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In this paper we shall give some generalizations for the notion of M-spaces introduced by K. Morita [8]. A space X is called an *M-space* if there exists a normal sequence $\{\mathfrak{U}_i\}$ of open coverings of X satisfying the following condition (M) below:

(M) If $\{K_i\}$ is a decreasing sequence of non-empty closed sets of X such that $K_i \subset \mathrm{St}(x_0, \mathfrak{U}_i)$ for each i and for a fixed point x_0 of X, then $\cap K_i \neq \phi$.

From condition (M) we obtain further a *condition* (M') (resp. (M_δ)) with the phrase "K_i is a closed set" replaced by "K_i is a zero set" (resp. "K_i is a closed G_δ-set") and we shall call a space X an *M'-space* (resp. *M_δ-space*) if X satisfies the condition (M′) (resp. (M_δ)). The class of M'-spaces contains all pseudocompact spaces and all M-spaces. There are properties for M'-spaces similar to those for M-spaces, for instance, an M'-space X has Morita's paracompactification μX which is obtained by K. Morita for M-spaces. Moreover, as a nice property of M'-space, any subspace of μX, containing X, is always an M'-space while this property does not hold in case X is an M-space.

For simplicity, we assume that all spaces are completely regular T_1-spaces and that mappings are continuous; we denote by βX and υX the Stone-Čech compactification and Hewitt realcompactification of a given space X respectively. For a mapping $\varphi: X \to Y$, the symbol Φ denotes the Stone extension of φ from βX onto βY. N is the set of all natural numbers. Other terminologies and notations will be used as in [3].

1. Characterization of M'-spaces.

Let φ be a mapping from X onto Y. φ is a *WZ-mapping* if $\mathrm{cl}_{\beta X}\varphi^{-1}(y) = \Phi^{-1}(y)$ for each $y \in Y$ [7] and φ is a Z (resp. Z_δ)*-mapping* if $\varphi(F)$ is closed for each zero set (resp. closed G_δ-set) F of X. A Z (resp. Z_δ)-mapping φ is a Z_p (resp. $Z_{\delta p}$)*-mapping* if $\varphi^{-1}(y)$ is pseudocompact for each $y \in Y$. A subset F of X is called a *relatively pseudocompact* if f is bounded on F for each $f \in C(X)$. A Z-mapping φ is said to be an *SZ-mapping* if $\varphi^{-1}(y)$ is relatively pseudocompact for each $y \in Y$.

K. Morita [8] has proved that X is an M-space if and only if there exists a quasi-perfect mapping φ from X onto some metric space Y where a closed mapping φ is called a *quasi-perfect* mapping if $\varphi^{-1}(y)$