73. On Infinitesimal Automorphisms of Siegel Domains

By Noboru TANAKA
Department of Mathematics, Kyoto University, Kyoto

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The aim of this note is to announce some theorems (Theorem 1–Theorem 4) concerning the Lie algebra $\mathfrak{g}$ of all infinitesimal automorphisms of a Siegel domain D of second kind. Theorems 3 and 4 enable us to calculate, in an algebraic manner, the Lie algebra $\mathfrak{g}$ on the basis of the Lie algebra $\mathfrak{g}_a$ of all infinitesimal affine automorphisms of D.

1. Let W^{-2} (resp. W^{-1}) be a real (resp. complex) vector space of finite dimension. We say that an open set V of W^{-2} is a convex cone in W^{-2} if it satisfies the following conditions:

1) $x+x'$, $\lambda x \in V$ for any $x, x' \in V$ and any real number $\lambda > 0$,

2) V contains no entire straight lines.

Given a convex cone V in W^{-2}, we say that a mapping F of $W^{-1}\times W^{-1}$ to W_c^{-2} (=the complexification of W^{-2}) is a V-hermitian form on W^{-1} if it satisfies the following conditions:

1) F is hermitian, i.e., $F(u, u')$ is complex linear with respect to the variable u, and $\overline{F(u, u')}=F(u', u)$,

2) F is V-positive definite, i.e., $F(u, u) \in \bar{V}$ for any u, and $F(u, u) \neq 0$ for any $u \neq 0$, where $\bar{V}$ denotes the closure of V in W^{-2}.

Suppose that we are given a convex cone V in W^{-2} and a V-hermitian form F on W^{-1}. We put $\tilde{W}=W_c^{-2}+W^{-1}$ and denote by z^{-2} (resp. z^{-1}) the projection of $\tilde{W}$ onto W_c^{-2} (resp. onto W^{-1}). Furthermore we define a mapping Φ of $\tilde{W}$ to W^{-2} by

$$\Phi(p)=\operatorname{Im} z^{-2}(p)-F(z^{-1}(p), z^{-1}(p)) \qquad (p \in \tilde{W}).$$

Then the domain $D=\Phi^{-1}(V)$ (=the inverse image of V by Φ) of $\tilde{W}$ is called the Siegel domain of second kind associated with the cone V and the V-hermitian form F (Pyatetski-Shapiro [2]). Let S be the real submanifold of $\tilde{W}$ defined by $\Phi=0$, i.e., $S=\Phi^{-1}(0)$. Then [2] has asserted that S is just the Silov boundary of the domain D with respect to an appropriate ring of holomorphic functions on D.

2. Hereafter we assume that D is affine homogeneous, that is, the group of all affine transformations of $\tilde{W}$ leaving D invariant acts transitively on D. A holomorphic vector field on D is called an infinitesimal automorphism of D if it generates a one parameter group of automorphisms of D or equivalently if it is complete as a vector field.