

132. On Infinitesimal Affine Automorphisms of Siegel Domains

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A non-empty open cone V in a finite dimensional vector space X over \mathbf{R} is called a *convex cone*, if it is convex and contains no straight lines. For example, the cone $\mathcal{P}(m, \mathbf{R})$ ($\mathcal{P}(m, \mathbf{C})$) of all positive-definite real symmetric (complex hermitian) matrices of degree m is a convex cone. For a convex cone V in X , an \mathbf{R} -bilinear map F on a finite dimensional vector space Y over \mathbf{C} into the complexification X^c of X is called a *V-hermitian function* if it is \mathbf{C} -linear with respect to the first variable and $F(u, v) = \overline{F(v, u)}$, where $z \rightarrow \bar{z}$ is the conjugation of X^c with respect to the real form X , and if it is V -positive-definite in the sense that $F(u, u) \in \bar{V}$ (the closure of V in X) and $F(u, u) = 0$ implies $u = 0$ for $u \in Y$. For a V -hermitian function F , the domain $D(V, F) = \{(z, u) \in X^c \times Y; \mathcal{I}_m z - F(u, u) \in V\}$ of $X^c \times Y$ is called a *Siegel domain* associated to V and F . A Siegel domain $D(V, F)$ in $X^c \times Y$ is called *irreducible* if Y is not the direct sum of two non-trivial subspaces which are mutually orthogonal with respect to F . For Siegel domains $D(V, F) \subset X^c \times Y$ and $D(V', F') \subset X'^c \times Y'$, an affine isomorphism φ of $X^c \times Y$ onto $X'^c \times Y'$ is called an *affine isomorphism* of $D(V, F)$ onto $D(V', F')$ if $\varphi(D(V, F)) = D(V', F')$. An affine isomorphism of a Siegel domain $D(V, F)$ onto itself is called an *affine automorphism* of $D(V, F)$. If the group of affine automorphisms of a Siegel domain is transitive on it the domain is said to be *homogeneous*.

In this note we shall state a theorem which reduces the classification of homogeneous Siegel domains with respect to affine isomorphism to the one of certain distributive algebras over \mathbf{R} and we shall describe the structure of the Lie algebra of the group of affine automorphisms of a homogeneous Siegel domain in terms of the above algebra.

A finite dimensional distributive algebra \mathfrak{C} over \mathbf{R} is called a *matrix algebra with involution* $*$ of rank $m+1$ if : 1) it is bigraded : $\mathfrak{C} = \sum_{1 \leq i, k \leq m+1} \mathfrak{C}_{ik}$, 2) $\mathfrak{C}_{ik}\mathfrak{C}_{kl} \subset \mathfrak{C}_{il}$, $\mathfrak{C}_{ik}\mathfrak{C}_{pq} = \{0\}$ if $k \neq p$, 3) $a \mapsto a^*$ is an involutive anti-automorphism of the algebra \mathfrak{C} , $\mathfrak{C}_{ik}^* = \mathfrak{C}_{ki}$, 4) if we put $n_{ik} = \dim \mathfrak{C}_{ik}$, we have $n_{ii} \neq 0$ for $1 \leq i \leq m+1$. Henceforce, a_{ik}, b_{ik}, \dots will always denote arbitrary elements of the subspace \mathfrak{C}_{ik} . A matrix